Performance characteristics of rectangular aerostatic thrust bearing by conformal mapping

Shang-Han Gao
School of Mechanical Engineering, Guangxi University of Science and Technology, Liuzhou, China, and
Sheng-Long Nong
Lushan College of Guangxi University of Science and Technology, Liuzhou, China

Abstract

Purpose – This paper aims to analyze the pressure distribution of rectangular aerostatic thrust bearing with a single air supply inlet using the complex potential theory and conformal mapping.

Design/methodology/approach – The Möbius transform is used to map the interior of a rectangle onto the interior of a unit circle, from which the pressure distribution and load carrying capacity are obtained. The calculation results are verified by finite difference method.

Findings – The constructed Möbius formula is very effective for the performance characteristics researches for the rectangular thrust bearing with a single air supply inlet. In addition, it is also noted that to obtain the optimized load carrying capacity, the square thrust bearing can be adopted.

Originality/value – The Möbius transform is found suitable to describe the pressure distribution of the rectangular thrust bearing with a single air supply inlet.

Keywords Load carrying capacity, Möbius transform, Conformal mapping, Pressure distribution, Rectangular aerostatic thrust bearing

Paper type Technical paper

Nomenclature

Notation

a	= rectangular length;
b	= rectangular length;
$c_1 \sim c_8$	= complex coefficient in conformal mapping function;
$d_1 \sim d_{16}$	= real and imaginary parts for complex coefficient $c_1 \sim c_8$;
D	= compressible flow region;
f	= numerical calculation parameter;
h	= film thickness;
H	= conformal mapping singular equation;
P	= pressure distribution;
P_i	= inlet pressure at inner boundary Γ_2;
P_0	= atmospheric pressure at outer boundary Γ_1;
r	= radius of concentric circle;
S	= bearing area;
u	= relative velocity along x direction;
v	= relative velocity along y direction;
w	= complex variable in unit circle;
x	= horizontal coordinate;
y	= vertical coordinate;
Δx	= segment element along x direction;
Δy	= segment element along y direction;
z	= complex variable in region D;
η	= kinematical viscosity;
ρ	= density;
Γ	= outer boundary;
Γ_2	= inner boundary;
θ	= mapping angular;
θ_a	= mapping angular;
θ_b	= mapping angular;
ξ_a	= rectangular length ratio;
ξ_b	= rectangular length ratio; and
λ	= mapping radius.

1. Introduction

Aerostatic bearings are widely used in super-precise and ultra-high speed mechanical systems because of their negligible friction and low heat generation. The main focus of researches on aerostatic bearings is the performance characteristics such as pressure distribution and load carrying capacity, which were represented by many researches, such as Li and Ding (2007), Chen *et al.* (2011) and Belforte *et al.* (2011).

The current issue and full text archive of this journal is available on Emerald Insight at: **www.emeraldinsight.com/0036-8792.htm**

Industrial Lubrication and Tribology
70/8 (2018) 1457–1475
Emerald Publishing Limited [ISSN 0036-8792]
[DOI 10.1108/ILT-08-2017-0238]

Received 19 August 2017
Accepted 28 January 2018

Fourka and Bonis (1997) analyzed the influence of feeding types on the performance of aerostatic bearings and described the numerical procedures based on the finite element method. The load capacity and stiffness were calculated with the integration of the pressure field, which was written in dimensionless form. The triangle finite elements and linear interpolation functions were adopted by Baksys and Ramonas (2009) to solve the non-stationary Reynolds' equation of pressure distribution in air cushion of a pneumatic track. Dependencies of air cushion pressure on system parameters were determined. Taking into account the equilibrium of the mass flow rate and the squeeze film effect, Chen and Lin (2002) adopted the Newmark integration method and the modified resistance network method to analyze the time dependent dynamic behaviors at each time step for designing an aerostatic rectangular bearing with X-shaped grooves. Yoshimoto *et al.* (2007) described the pressure distribution in the bearing clearance of circular aerostatic thrust bearings with a single air supply inlet. The flow field was calculated by using CFD software, which can solve the Navier–Stokes equations directly. In addition, the numerical results were verified by the experimental data. Nishio *et al.* (2011) considered the aerostatic annular thrust bearings with feedholes of less than 0.05 mm in diameter, and investigated the static and dynamic characteristics of these bearings by using the finite difference method. The discharge coefficients of small feedhole were determined also by using CFD and verified experimentally.

To study the applicability of the Reynolds equation and the precise calculation of the flow field at large gas film clearance for a thrust aerostatic bearing, Yu and Ma (2010) adopted both the finite difference method and the finite volume method to calculate the two-dimensional flow field. For the finite difference method, the Newton iteration method for two-dimensional Reynolds equation of thrust aerostatic bearings was deduced, and the Chebyshev accelerated over-relaxation iterative method was introduced to improve the calculation efficiency. For the finite volume method based on the Fluent Software, the structured and partial refinement grids were applied and the mixture model of laminar and turbulent models were built and solved by the SIMPLE algorithm. To capture turbulent structures and fluctuations, Zhu *et al.* (2013) used large eddy simulation method to numerically calculate the transient flow field in the bearing clearance. Vortex structures and pressure fluctuation in the bearing clearance were captured. Relationship between the pressure fluctuation and bearing vibration was established based on the simulation results and experimentally measured vibration strength. For the sake of predicting accurately the characteristics of supersonic flow in externally pressurized spherical air bearings under large clearance and high air supply pressure, Chang and Jeng (2014) proposed a modified particle swarm optimization algorithm to optimize a double-pad aerostatic bearing. The modified particle swarm optimization algorithm has a global search capability and high efficiency to optimize a problem with several design variables and that the mutation can provide an avenue for particles to escape from a local optimal value. A surrogate model of the discharge coefficient of an orifice-type restrictor based on the orifice diameter and film thickness was also built by using the artificial neural networks.

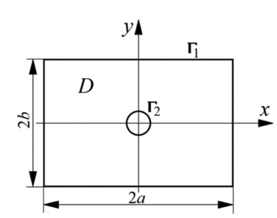
Since the creation of the flow field equations inside air bearing clearance, scientists were committed to search the solutions. Many studies have been made to investigate the static and dynamic characteristics and performance of aerostatic bearings by using the numerical or experimental methods. With the application of numerical methods such as finite differences and finite elements methods, the numerical solutions are reliable only when a large number of nodal points are considered. This will eventually lead to prohibitive computational cost owing to the use of iterations. Conversely, adopting approximate but accurate analytic procedures leads to simplified formulation and acceptable results. Several literatures attempted to look for the analytic solutions for the externally pressurized bearings. The Stokesian viscous fluid lubricants in an externally pressurized hydrostatic 2:1 ellipse bearing were studied by Bilal (2012) with complex potential theory and conformal mapping. The Navier–Stokes' equations in three dimensions were reduced to the potential equation in two dimensions associated with Dirichlet boundary conditions. Applying the Riemann's mapping theorem and Jacobian elliptic functions, the mapping transformation was established. Balcerzak and Raynor (1962) computed the performance of hydrostatic bearing pads in the form of n-sided regular polygons with a central oil supply hole by conformal mapping method.

For accurately computing the pressure distribution for the rectangular aerostatic thrust bearing with a single air supply inlet, the Möbius transform is adopted to map the interior of rectangle onto the interior of the unit circle in this paper, from which, the load carrying capacity is analyzed. The rest of this paper is organized as follows. Section 2 explains the formulation of compressed air flow field. Section 3 constructs the conformal mapping function with Möbius transform. Sections 4 and 5, respectively, detail the mapping angle and inlet radius. Sections 6 and 7, respectively, illustrate the pressure distribution and load carrying capacity of the rectangular bearing. The availability of the conformal mapping function proposed is verified by numerical solution in Section 8, and further researches of the load carrying capacity are described in Section 9. Finally, conclusions are drawn from the above results.

2. Formulation of compressible air

A typical rectangular aerostatic thrust bearing with a single air supply inlet illustrates in Figure 1. The domain D is bounded by two closed contours, Γ_1 and Γ_2. Where Γ_1 is a rectangle and Γ_2 is a concentric circle of radius with $r = 0.05$. $2a$ and $2b$ are the long side and short side of the rectangle, respectively. The compressible fluid flow occurs in D in such a way that the

Figure 1 Bearing configuration



Industrial Lubrication and Tribology
Volume 70 · Number 8 · 2018 · 1457–1475

atmospheric pressure P_0 retains at all the points of Γ_1, while the constant value P_1 is assumed all over the points of Γ_2.

With the assumptions of perfect gas for air and the isothermal laminar flow situated in bearing clearance, the non-dimensional Reynolds equation which is derived from Navier–Stokes equations, and can be expressed in the two dimensional Cartesian coordinates system:

$$\frac{\partial}{\partial x}\left(h^3 p \frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(h^3 p \frac{\partial p}{\partial y}\right) = 12\eta \frac{\partial(\rho h)}{\partial t} + 6\eta\left[u\frac{\partial(\rho h)}{\partial x} + v\frac{\partial(\rho h)}{\partial y}\right] \quad (1)$$

The above Reynolds equation can be further simplified by introducing the following list:

- The laminar flow of the boundary layer is fully developed steady state, and the transient term can be ignored, that is $\frac{\partial(\rho h)}{\partial t} = 0$.
- The aerostatic thrust bearings is stationary relatively to the pedestal, that is $u = 0$ and $v = 0$.
- The flow in the gas film clearance is steady, and the gas film thickness remains constant along the x and y directions, that is $\frac{\partial h}{\partial x} = 0$ and $\frac{\partial h}{\partial y} = 0$.

On the basis of the above assumptions, the compressible fluid Reynolds equation for rectangular thrust bearing can be simplified as follows:

$$\frac{\partial^2 p^2}{\partial x^2} + \frac{\partial^2 p^2}{\partial y^2} = 0 \quad (2)$$

The boundary conditions are given by:

$$P = P_1 \; x + iy \in \Gamma_2$$
$$P = P_0 \; x + iy \in \Gamma_1$$

where $P_1 = 1$ and $P_0 = 0.2$.

3. Möbius transform

The Riemann mapping theorem states that any simply connected domain can be mapped conformably onto the interior of the unit circle. Using the complex potential theory and conformal mapping, the function that maps the interior of a rectangle onto the interior of a unit circle can be deduced (Figure 2).

To construct the suitable conformal mapping function, the corresponding matching points that map the rectangle to the unit circle should be recognized first. Obviously, the mapping relations can be established between the four points (Nos 1, 3, 5 and 7) located on the coordinate axes x and y of the rectangle and the corresponding four points on coordinate axes η and ζ of the unit circle (Figure 3). To the four points at the rectangle corner, Nos 2, 4, 6 and 8, it is difficult to determine the corresponding matching points on the unit circle directly. Therefore, given the corner point No. 2 maps to the unit circle with angle of θ, the other corner points, Nos 4, 6 and 8, are assumed map to the points with angles of $180 - \theta$, $180 + \theta$ and $360 - \theta$, respectively, by taking into account the symmetry of the rectangle. The corresponding matching points from the rectangle to the unit circle are summarized in Table I:

Using the Möbius transform, the conformal mapping function is constructed as follows:

$$w = c_1\left(\frac{z}{z + c_2}\right)\left(\frac{z + c_3}{z + c_4}\right)\left(\frac{z + c_5}{z + c_6}\right)\left(\frac{z + c_7}{z + c_8}\right) \quad (3)$$

The unknown complex potential parameters $c_1 - c_8$ in equation (3) are determined by the mapping relationships involving the 8 matching points (showed in Table I), which are written as follows:

$$c_1 = d_1 + id_2$$
$$c_2 = d_3 + id_4$$
$$c_3 = d_5 + id_6$$
$$c_4 = d_7 + id_8$$
$$c_5 = d_9 + id_{10}$$
$$c_6 = d_{11} + id_{12}$$
$$c_7 = d_{13} + id_{14}$$
$$c_8 = d_{15} + id_{16}$$

Figure 2 Mapping domains

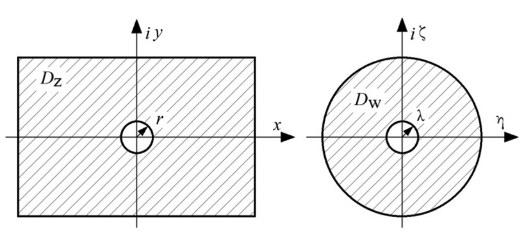


Figure 3 Mapping from rectangle to unit circle

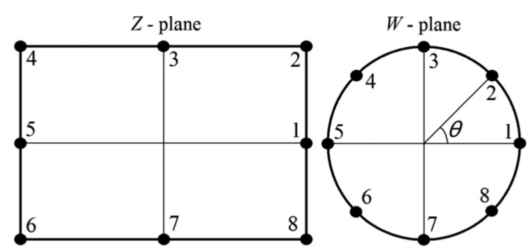


Table I Corresponding matching points

Matching points	1	2	3	4
Z-plane	a, 0	a, b	0, b	$-a$, b
W-plane	1, 0	cos(θ), sin(θ)	0, 1	−cos(θ), sin(θ)
Matching points	5	6	7	8
Z-plane	$-a$, 0	$-a$, $-b$	0, $-b$	a, $-b$
W-plane	−1, 0	−cos(θ), −sin(θ)	0, −1	cos(θ), −sin(θ)

Note: Where $\theta = 0 \sim \frac{\pi}{2}$

Industrial Lubrication and Tribology
Volume 70 · Number 8 · 2018 · 1457–1475

Introducing the characteristic function:

$$H = w(z + c_2)(z + c_4)(z + c_6)(z + c_8) - c_1 z(z + c_3)(z + c_5)(z + c_7) = 0 \tag{4}$$

The real and imaginary functions of H are given as follows:

$$\begin{cases} Re(H) = 0 \\ Im(H) = 0 \end{cases} \tag{5}$$

Substituting the mapping relationships into equation (5) yields 16 characteristic equations (showed in Appendix 1) referred to 16 real numbers $d_1 - d_{16}$. By resolving the 16 characteristic equations, the mapping function [equation (3)] from the interior of rectangle to the interior of unit circle is obtained.

4. Mapping angle θ

By introducing the parameter of aspect ratio $\xi_a = \frac{a}{b}$, Figure 4 illustrates the conformal mapping graphics from the interior of rectangle to the interior of unit circle with the variation of ξ_α for $b = 1$. These graphics show that the Formula (3) can be used to build a mapping relationship between the rectangle and the unit circle.

In addition, it is noted that the rectangular corner point No. 2 maps to different point on the unit circle for different aspect ratio ξ_a. As $\xi_a = 1$, it can be observed that $\theta = 45°$. Increasing ξ_a, the mapping angle θ decreases. For example, $\xi_a = 1.5$ produces $\theta = 21.6°$ and $\xi_a = 2$ produces $\theta = 9.9°$. The relationship between the mapping angle and the aspect ratio is illustrated on Figure 5. And the corresponding fitting curves in Figure 6 show that as $\xi_\alpha = \xi_b$, $\theta_a = 90^\circ - \theta_b$, which can be explained from the rectangular symmetry. Taking the feature of symmetry, just the situation of $a > b$ is considered in the following sections, and ξ_a is selected as the variation parameter to research the performance characteristics of the rectangular bearing.

5. Mapping concentric circle

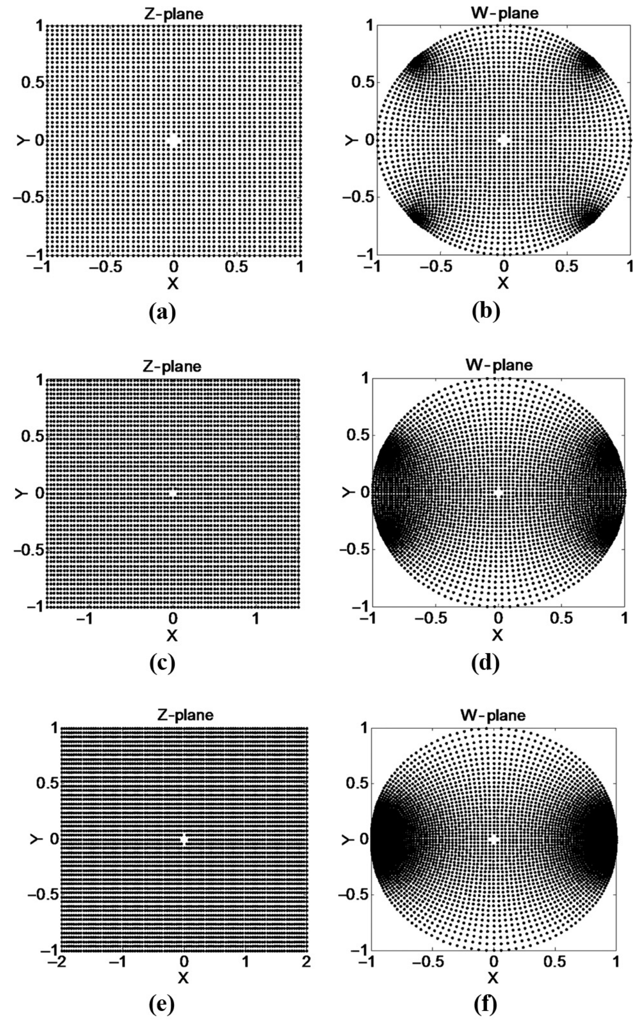
Adopting the constructed Möbius transform, the air supply inlet with a radius of r inside a rectangle is mapped to the concentric circle with a radius of λ inside the unit circle (Figure 7).

The radius of mapped concentric circle λ inside the unit circle decreases (Figure 8) with the increase in ξ_a at $b = 1$. It is also noted from Figure 9 that λ decreases with the increase of bearing area S. And at the same bearing surface area situations, the increase of length ratio ξ_a improves λ.

6. Pressure distribution

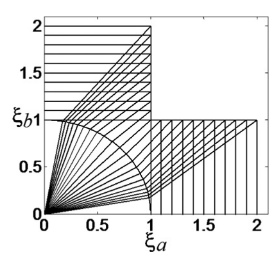
For the simplified Reynolds equation (2), which governs the compressible flow field in the clearance space for a unit circle with a concentric circle of radius λ, the steady pressure distribution is deduced by integrating the Reynolds equation and imposing the boundary conditions:

Figure 4 Conformal mapping from rectangle to unit circle with b = 1



Notes: (a) z-plane, $\xi_a = 1.0$; (b) *w*-plane, $\xi_a = 1.0$; (c) z-plane, $\xi_a = 1.5$; (d) *w*-plane, $\xi_a = 1.5$; (e) z-plane, $\xi_a = 2.0$; (f) *w*-plane, $\xi_a = 2.0$

Figure 5 Mapping angle



$$p = \sqrt{Re\left[(p_1{}^2 - p_0{}^2)\frac{ln\ w}{ln\ \lambda} + p_0{}^2\right]} \tag{6}$$

Substituting the Möbius transform [equation (3)] into equation (6), the pressure distributions of the rectangular bearing with a single air supply inlet are obtained and

Figure 6 Curves of mapping angle with length ratio

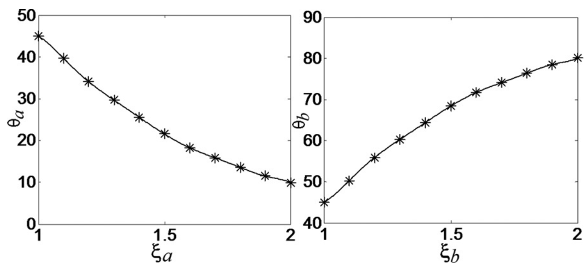
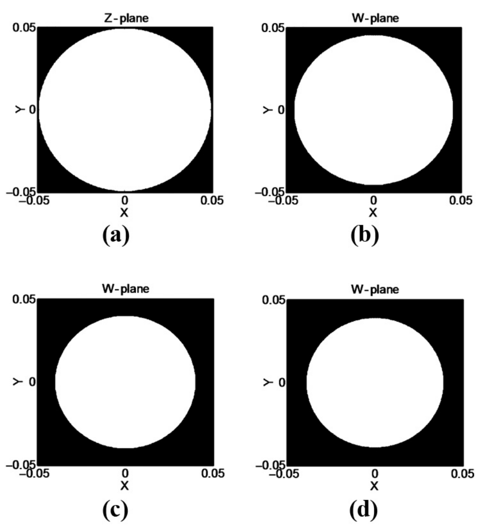
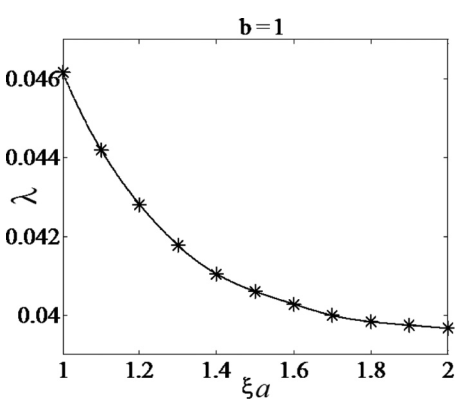


Figure 7 Concentric circles



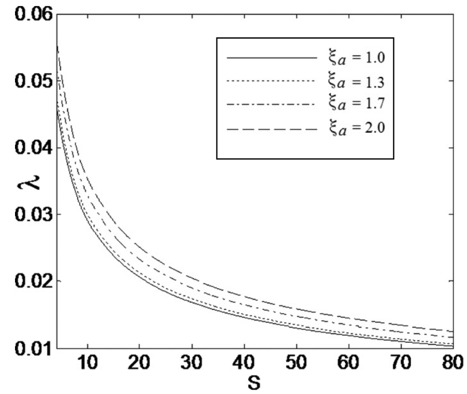
Notes: (a) $r = 0.05$ mm in z-plane; (b) $\xi_a = 1.0$, in w-plane; (c) $\xi_a = 1.5$, in w-plane; (d) $\xi_a = 2.0$, in w-plane

Figure 8 Radius of mapped concentric circle with ξ_a



illustrated in Figure 10 for different aspect ratio situations. From Figure 10, the inlet pressure $P_1 = 1.0$ within the radius of air supply inlet, and the atmospheric pressure $P_0 = 0.2$ at the bearing outer edge, which are consistent with the given boundary conditions.

Figure 9 Mapped concentric circle with rectangular area



7. Load carrying capacity

Normally, the bearing load capacity is calculated by integrating the pressure profile inside the bearing clearance as follows:

$$F = \iint \mathrm{pdxdy} \tag{7}$$

In this article, equation (7) is very hard to be resolved analytically owing to the nonlinear expression of pressure distribution [equation (6)]. Numerical integrating method [equation (8)] is therefore adopted to substitute equation (7) to resolve the load carrying capacity:

$$F = \sum_{\mathrm{i}=1}^{\mathrm{n}} p_i \Delta s_i \tag{8}$$

The load carrying capacity calculating results are showed in Figure 11, which is observed improve with the increase of ξ_a. It is clear that, fixed the length of one side of the rectangle, the larger rectangular area brings the higher load carrying capacity.

8. Verifying the conformal mapping method

For verifying the proposed conformal mapping method, the finite difference numerical technique is used to solve the load carrying capacity of the bearing. From the comparison results between the analytic method and the numerical technique, the validity of the Möbius transform can be verified.

For the compressible flow field partial differential equation (2), letting $f = p^2$, yields the following:

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial \mathrm{y}^2} = 0 \tag{9}$$

The boundary conditions are as follows:

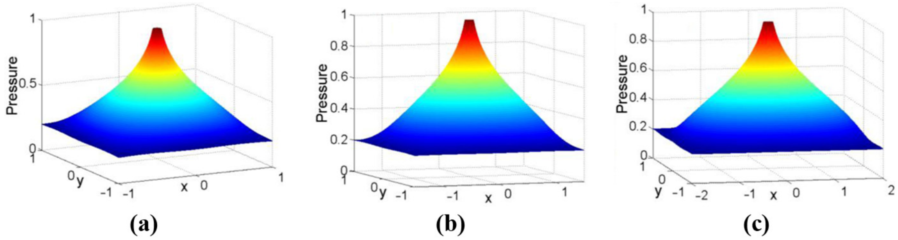
$$f = f_1\, x + iy \in \Gamma_2$$

$$f = f_o\, x + iy \in \Gamma_1$$

where, $f_1 = 1$ and $f_o = 0.04$

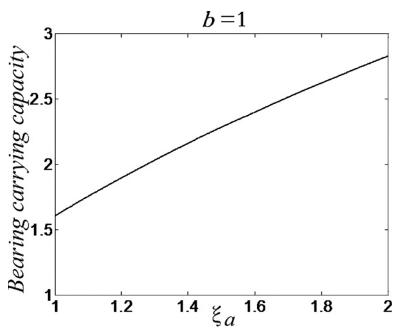
The second derivatives for the partial differential terms in equation (9) could be approximated by the follow equations:

Figure 10 Pressure distribution diagrams for $b = 1$



Notes: (a) $\xi_a = 1.0$; (b) $\xi_a = 1.5$; (c) $\xi_a = 2.0$

Figure 11 Curve of load carrying capacity with ξ_a for $b = 1$



$$\frac{\partial^2 f}{\partial x^2} = \frac{f_{i+1,j} - 2f_{i,j} + f_{i-1,j}}{(\Delta x)^2} \tag{10}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{f_{i,j+1} - 2f_{i,j} + f_{i,j-1}}{(\Delta y)^2} \tag{11}$$

Substituting equations (10) and (11) into equation (9) gives:

$$f_{i,j} = \frac{(\Delta y)^2 f_{i+1,j} + (\Delta y)^2 f_{i-1,j} + (\Delta x)^2 f_{i,j+1} + (\Delta x)^2 f_{i,j-1}}{2(\Delta y)^2 + 2(\Delta x)^2} \tag{12}$$

and:

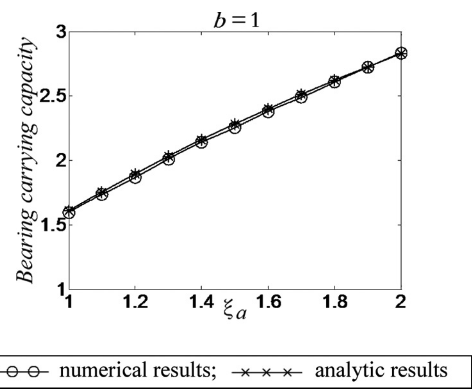
$$p_{i,j} = \sqrt{f_{i,j}} \tag{13}$$

Figure 12 illustrates the comparison results between the conformal mapping method and the numerical technique. The conformity is found excellent, which indicates that the proposed conformal mapping formula is appropriate for the pressure distribution researches of the rectangular bearing with a single air supply inlet.

9. Further researches on load carrying characteristics

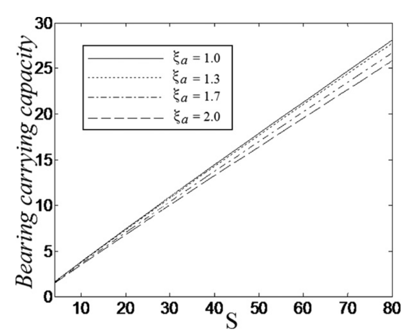
In this section, the loading characteristics of the rectangular bearing are investigated by the effects of the aspect ratio ξ_a and bearing area S.

Figure 12 Comparison between the analytic and numerical results



As the bearing area S increase, the load carrying capacity improves (Figure 13). At the same bearing area conditions, the larger aspect ratio decreases the load carrying capacity. It has been discovered from Figure 9 that, at the same bearing area situation, the larger length ratio ξ_a produces the larger radius of mapped concentric circle λ. For λ locates at the denominator position of the analytic pressure distribution equation (6). Increasing λ likely decreases the pressure value inside the bearing clearance, and accordingly reduces the load carrying capacity value. Combining the analysis from equation (6) and the results in Figures 9 and 13, it can be concluded that the bigger the rectangular length ratio takes, the smaller the bearing's load carrying capacity, at the same bearing area situations. Therefore, to get the maximum load carrying capacity for the rectangular thrust bearing, the aspect ratio of the rectangle should be given as $\xi_a = 1$, and that exactly is a square bearing.

Figure 13 Curves of the load carrying capacity with ξ_a and S



10. Conclusion

Owing to the virtue of low friction and high accuracy, the aerostatic bearings are widely used in super-precise and ultra-high speed mechanical systems. The determination of performance characteristics for the aerostatic bearing system is, therefore, very essential. The complex potential theory and Möbius transform are adopted in this work to satisfy the boundary conditions of the rectangular bearing, from which, the pressure distribution and load carrying capacity are obtained. The analytic results are compared with the results of the finite difference method, and conformity is found excellent, which shows that the Möbius transform constructed in this paper is suitable to describe the pressure distribution of the rectangular thrust bearing with a single air supply inlet. In addition, it is also noted that to obtain the optimized bearing carrying capacity, the square thrust bearing can be adopted.

References

Baksys, B. and Ramonas, A. (2009), "Simulation of body vibrations on an incompressible air cushion", *Acta Mechanica*, Vol. 203 Nos 1/2, pp. 77-88.

Balcerzak, M.J. and Raynor, S. (1962), "Solutions for several shapes of externally pressurized hydrostatic thrust bearing", *Applied Scientific Research*, Vol. 11 No. 2, pp. 189-217.

Belforte, G., Colombo, F., Raparelli, T., Trivella, A. and Viktorov, V. (2011), "Comparison between grooved and plane aerostatic thrust bearings: static performance", *Meccanica*, Vol. 46 No. 3, pp. 547-555.

Bilal, M.A.M. (2012), "Performance characteristics of an elliptic hydrostatic bearing and comparative analysis based on Stokes' conditions", *Acta Mechanica*, Vol. 223 No. 6, pp. 1187-1198.

Chang, S.H. and Jeng, Y.R. (2014), "A modified particle swarm optimization algorithm for the design of a double-pad aerostatic bearing with a pocketed orifice-type restrictor", *Journal of Tribology*, Vol. 136 No. 2, pp. 021701-1-7.

Chen, M.F. and Lin, Y.T. (2002), "Static behavior and dynamic stability analysis of grooved rectangular aerostatic thrust bearings by modified resistance network method", *Tribology International*, Vol. 35 No. 5, pp. 329-338.

Chen, X.D., Chen, H., Luo, X., Ye, Y.X., Hu, Y.T. and Xu, J. Q. (2011), "Air vortices and nano-vibration of aerostatic bearings", *Tribology Letters*, Vol. 42 No. 2, pp. 179-183.

Fourka, M. and Bonis, M. (1997), "Comparison between externally pressurized gas thrust bearings with different orifice and porous feeding systems", *Wear*, Vol. 210 Nos 1/2, pp. 311-317.

Li, Y.T. and Ding, H. (2007), "Influences of the geometrical parameters of aerostatic thrust bearing with pocketed orifice-type restrictor on its performance", *Tribology International*, Vol. 40 No. 7, pp. 1120-1126.

Nishio, U., Somaya, K. and Yoshimoto, S. (2011), "Numerical calculation and experimental verification of static and dynamic characteristics of aerostatic thrust bearings with small feedholes", *Tribology International*, Vol. 44 No. 12, pp. 1790-1795.

Yoshimoto, S., Yamamoto, M. and Toda, K. (2007), "Numerical calculations of pressure distribution in the bearing clearance of circular aerostatic thrust bearings with a single air supply inlet", *Journal of Tribology*, Vol. 129 No. 2, pp. 384-390.

Yu, H.C. and Ma, W.Q. (2010), "Study on two different calculation methods for thrust aerostatic bearings with single supply hole at large gas film clearance", *Proceedings of the 2010 IEEE International Conference on Mechatronics and Automation August, Xi'an*, Vols 4/7, pp. 53-58.

Zhu, J.C., Chen, H. and Chen, X.D. (2013), "Large eddy simulation of vortex shedding and pressure fluctuation in aerostatic bearings", *Journal of Fluids and Structures*, Vol. 40, pp. 42-51.

Appendix 1

$$
\begin{aligned}
&-ad_8d_{12}d_{15}+d_3ad_{11}d_{15}+ad_7d_{11}d_{15}-ad_7d_{12}d_{16}-ad_8d_{16}d_{11}\\
&+d_7a^2d_{15}+d_3a^2d_{15}-a^2d_{12}d_{16}-d_4d_{12}a^2-d_8d_{16}a^2\\
&+d_2d_{10}a^3+d_3d_{11}a^2-d_4d_8a^2+d_3d_7a^2+a^2d_{11}d_{15}+d_7d_{11}a^2\\
&-d_8d_{12}a^2-d_4d_{16}a^2+a^4-d_3d_8d_{16}d_{11}-d_3d_8d_{12}a\\
&-d_3d_8d_{12}d_{15}-d_4ad_{16}d_{11}-d_4ad_{12}d_{15}-d_4d_7d_{16}a\\
&-d_4d_7d_{16}d_{11}-d_4d_7d_{12}a-d_4d_7d_{12}d_{15}-d_4d_8ad_{15}\\
&-d_4d_8d_{11}a-d_3ad_{12}d_{16}+d_3d_7ad_{15}+d_3d_7d_{11}a+d_3d_7d_{11}d_{15}\\
&-d_3d_7d_{12}d_{16}-d_3d_8d_{16}a+d_2a^2d_{14}d_9-d_4d_8d_{11}d_{15}\\
&+d_4d_8d_{12}d_{16}-d_1a^2d_9d_{13}+d_1a^2d_{10}d_{14}-d_1d_5a^2d_{13}\\
&-d_1d_5d_9a^2+d_1d_6d_{14}a^2+d_1d_6d_{10}a^2+ad_1d_5d_{10}d_{14}\\
&-ad_1d_5d_9d_{13}+d_2a^2d_{10}d_{13}+d_2d_5d_{14}a^2+d_2d_5d_{10}a^2\\
&+d_2d_6a^2d_{13}+d_2d_6d_9a^2+ad_1d_6d_{14}d_9+ad_1d_6d_{10}d_{13}\\
&+ad_2d_5d_{14}d_9+ad_2d_5d_{10}d_{13}+ad_2d_6d_9d_{13}-ad_2d_6d_{10}d_{14}\\
&-d_1d_9a^3-d_1a^3d_{13}+d_2d_6a^3-d_1d_5a^3+d_2d_{14}a^3+a^3d_{15}\\
&+d_{11}a^3+d_7a^3+d_3a^3-d_1a^4=0
\end{aligned}
\tag{B1}
$$

$$
\begin{aligned}
&ad_7d_{12}d_{15}+ad_8d_{11}d_{15}-ad_8d_{12}d_{16}+d_8d_{11}a^2-d_1d_6a^3\\
&+a^2d_{16}d_{11}+ad_7d_{16}d_{11}-d_1a^2d_{10}d_{13}+d_3ad_{16}d_{11}\\
&+d_3ad_{12}d_{15}+d_3d_7d_{16}a+d_3d_7d_{16}d_{11}+d_3d_7d_{12}a+d_3d_7d_{12}d_{15}\\
&+d_3d_8ad_{15}+d_3d_8d_{11}a-d_3d_8d_{12}d_{16}+d_4ad_{11}d_{15}-d_4ad_{12}d_{16}\\
&+d_4d_7ad_{15}+d_4d_7d_{11}a+d_4d_7d_{11}d_{15}-d_4d_7d_{12}d_{16}-d_4d_8d_{16}a\\
&-d_4d_8d_{16}d_{11}-d_4d_8d_{12}a-d_4d_8d_{12}d_{15}-d_1a^2d_{14}d_9\\
&+d_3d_8d_{11}d_{15}+d_7d_{12}a^2+d_3d_8a^2+d_3d_{16}a^2+d_4a^2d_{15}\\
&+a^2d_{12}d_{15}+d_3d_{12}a^2+d_4d_7a^2+d_7d_{16}a^2-d_2d_9a^3\\
&-d_2a^3d_{13}+d_8a^2d_{15}-d_1d_{10}a^3-d_1d_{14}a^3+d_4d_{11}a^2-d_2d_5a^3\\
&-d_1d_5d_{14}a^2-d_1d_5d_{10}a^2-d_1d_6a^2d_{13}-d_1d_6d_9a^2
\end{aligned}
$$

$$- d_2a^2d_9d_{13} + d_2a^2d_{10}d_{14} - d_2d_5a^2d_{13} - d_2d_5d_9a^2 + d_2d_6d_{14}a^2 + ad_2d_6d_{10}d_{13} - ad_1d_6d_9d_{13} + ad_1d_6d_{10}d_{14} - ad_2d_5d_9d_{13} + ad_2d_5d_{10}d_{14} + ad_2d_6d_{14}d_9 - ad_1d_5d_{14}d_9 - ad_1d_5d_{10}d_{13} + d_2d_6d_{10}a^2 + d_{16}a^3 + d_{12}a^3 + d_8a^3 + d_4a^3 - d_2a^4 = 0 \tag{B2}$$

$$- d_1d_5a^2d_{13} + 2sin(\theta)ad_8d_{12}b + cos(\theta)d_8b^2d_{16} - 2sin(\theta)d_3d_7ab + sin(\theta)d_3bd_{12}d_{16} + sin(\theta)d_4d_7bd_{16} + sin(\theta)d_3d_{12}b^2 + sin(\theta)d_3b^2d_{16} - d_1b^2d_{10}d_{14} - d_1d_6b^2d_{14} + 2d_1ad_5bd_{14} + sin(\theta)d_4d_8bd_{15} + 2sin(\theta)d_4ad_{12}b - sin(\theta)d_4d_{11}a^2 - 3sin(\theta)a^2d_{11}b + d_1bd_5d_{10}d_{13} + d_1bd_6d_9d_{13} - cos(\theta)d_3b^2d_{15} - cos(\theta)d_7d_{11}b^2 - d_2bd_5d_{10}d_{14} - sin(\theta)d_3bd_{11}d_{15} - sin(\theta)d_7d_{16}a^2 + 3d_1ab^2d_{13} + d_1d_5d_9b^2 + sin(\theta)bd_8d_16d_{11} - sin(\theta)d_3d_7bd_{15} - 2d_2abd_{10}d_{14} - 3sin(\theta)a^2bd_{15} + d_1d_5b^2d_{13} + d_1b^2d_9d_{13} - 2sin(\theta)d_3ad_{11}b - 3sin(\theta)d_7a^2b + 3sin(\theta)ab^2d_{16} + 3sin(\theta)ad_{12}b^2 - d_1b^4 + sin(\theta)d_4d_{11}b^2 + sin(\theta)d_4b^2d_{15} + 3d_1a^2bd_{14} + sin(\theta)bd_7d_{12}d_{16} + d_2d_{14}a^3 - d_1d_9a^3 - 6cos(\theta)a^2b^2 + d_2d_{10}a^3 + cos(\theta)d_7a^3 - d_1a^3d_{13} + d_2d_6a^3 + cos(\theta)d_{12}b^3 - d_1d_5a^3 + cos(\theta)d_3a^3 + cos(\theta)a^3d_{15} + cos(\theta)d_8b^3 + cos(\theta)d_{11}a^3 + sin(\theta)d_4d_7d_{12}b + cos(\theta)b^3d_{16} - 4sin(\theta)a^3b + cos(\theta)d_4b^3 - sin(\theta)d_{16}a^3 - sin(\theta)d_{12}a^3 + 6d_1a^2b^2 - sin(\theta)d_4a^3 - sin(\theta)d_8a^3 + 4sin(\theta)ab^3 - sin(\theta)bd_7d_{11}d_{15} - 2sin(\theta)d_3abd_{15} + cos(\theta)d_4b^2d_{16} + sin(\theta)b^2d_{16}d_{11} - sin(\theta)d_3d_7d_{11}b + cos(\theta)d_8d_{12}b^2 - 3cos(\theta)d_4a^2b + cos(\theta)a^2d_{11}d_{15} + 3d_2a^2d_9b + 3d_2a^2bd_{13} + 3d_2d_5a^2b - sin(\theta)d_4a^2d_{15} - d_1a^2d_9d_{13} - d_1bd_6d_{10}d_{14} - sin(\theta)d_8d_{11}a^2 - sin(\theta)d_3d_8a^2 - cos(\theta)a^2d_{12}d_{16} + cos(\theta)d_7a^2d_{15} + cos(\theta)d_7d_{11}a^2 - cos(\theta)d_8d_{16}a^2 - cos(\theta)d_8d_{12}a^2 + cos(\theta)d_3d_{11}a^2 + cos(\theta)d_3d_7a^2 - cos(\theta)d_4d_{16}a^2 - cos(\theta)d_4d_{12}a^2 - cos(\theta)d_4d_8a^2 - 3cos(\theta)a^2bd_{16} - 3cos(\theta)a^2d_{12}b + 2d_2ad_5bd_{13} + sin(\theta)d_4bd_{12}d_{15} + cos(\theta)d_3a^2d_{15} - cos(\theta)d_3d_{11}b^2 + cos(\theta)b^4 - d_2d_5d_{10}b^2 - ad_1d_5d_9d_{13} + ad_1d_5d_{10}d_{14} + ad_1d_6d_{14}d_9 + ad_1d_6d_{10}d_{13} + ad_2d_5d_{14}d_9 + ad_2d_5d_{10}d_{13} + ad_2d_6d_9d_{13} - ad_2d_6d_{10}d_{14} - d_1a^4 - 3d_2ab^2d_{14} - 3d_2ad_{10}b^2 + 3d_1ad_9b^2 + d_1bd_5d_{14}d_9 - d_2b^2d_{14}d_9 - d_2b^2d_{10}d_{13} + sin(\theta)d_7d_{12}b^2 + sin(\theta)d_3b^3 + sin(\theta)d_7b^3 + sin(\theta)d_{11}b^3 + sin(\theta)b^3d_{15} - d_1d_{10}b^3 - d_1d_6b^3 - d_1b^3\mathrm{d}_{14} + 4d_2a^3b - 4d_2ab^3 - d_2d_5b^3 - d_2d_9b^3 - d_2b^3d_{13} + sin(\theta)d_4bd_{16}d_{11} + cos(\theta)b^2d_{12}d_{16} - d_2d_6d_9b^2 - d_2d_6b^2d_{13} + \cos(\theta)ad_7d_{11}d_{15} + d_2bd_5d_9d_{13} + sin(\theta)bd_8d_{12}d_{15} - cos(\theta)ad_7d_{12}d_{16} - cos(\theta)ad_8d_{16}d_{11} - cos(\theta)ad_8d_{12}d_{15} + cos(\theta)d_3ad_{11}d_{15} - cos(\theta)d_3ad_{12}d_{16} + cos(\theta)d_3d_7ad_{15} + cos(\theta)d_3d_7d_{11}a + cos(\theta)d_3d_7d_{11}d_{15} + 2d_2abd_9d_{13} - cos(\theta)d_4\mathrm{a}d_{12}d_{15} - cos(\theta)d_4ad_{16}d_{11} - cos(\theta)d_3d_7d_{12}d_{16} - cos(\theta)d_3d_8d_{16}a - cos(\theta)d_3d_8d_{16}d_{11} - cos(\theta)d_3d_8d_{12}a - cos(\theta)d_3d_8d_{12}d_{15} - d_2bd_6d_{10}d_{13} + sin(\theta)b^2d_{12}d_{15} + sin(\theta)d_8d_{11}b^2 - cos(\theta)d_4d_7d_{16}a - cos(\theta)d_4d_7d_{16}d_{11} - cos(\theta)d_4d_7d_{12}a - cos(\theta)d_4d_7d_{12}d_{15} - cos(\theta)d_4d_8ad_{15} + 2sin(\theta)d_4d_8ab + sin(\theta)d_3d_8b^2 + sin(\theta)d_7b^2d_{16} + d_2d_6d_9a^2 + d_2d_5d_{10}a^2 + d_2d_6a^2d_{13} - cos(\theta)d_4d_8d_{11}a - cos(\theta)d_4d_8d_{11}d_{15} - cos(\theta)b^2d_{11}d_{15} + cos(\theta)d_4d_8d_{12}d_{16} - 2cos(\theta)ad_7bd_{16} - 2cos(\theta)ad_7d_{12}b - 2cos(\theta)abd_{16}d_{11} - 2cos(\theta)abd_{12}d_{15} - 2cos(\theta)ad_8d_{11}b - sin(\theta)d_7d_{12}a^2 - sin(\theta)a^2d_{16}d_{11} - 2cos(\theta)ad_8bd_{15} - 2cos(\theta)d_3abd_{16} - 2cos(\theta)d_3ad_{12}b - cos(\theta)d_3d_7bd_{16} - cos(\theta)d_3d_7d_{12}b - cos(\theta)d_3bd_{16}d_{11} - cos(\theta)d_3bd_{12}d_{15} - d_2bd_6d_{14}d_9 + 3sin(\theta)ad_8b^2 - 3sin(\theta)d_3a^2b - cos(\theta)bd_7d_{16}d_{11} - 2cos(\theta)d_3d_8ab - cos(\theta)d_3d_8d_{11}b - cos(\theta)d_3d_8bd_{15} + cos(\theta)bd_8d_{12}d_{16} - cos(\theta)bd_7d_{12}d_{15} - cos(\theta)bd_8d_{11}d_{15} - 3d_2ad_6b^2 - d_2d_5b^2d_{14} + d_2d_5d_{14}a^2 - sin(\theta)d_4d_7a^2 - d_1d_6d_{10}b^2 + 2d_1ad_6bd_{13} + 2d_1abd_{10}d_{13} + cos(\theta)a^4 - cos(\theta)d_7b^2d_{15} - cos(\theta)d_3d_7b^2 + sin(\theta)d_3d_8d_{12}b + 2d_1ad_6d_9b + d_2a^2d_{10}d_{13} + cos(\theta)d_4d_{12}b^2 + cos(\theta)d_4d_8b^2 - 3cos(\theta)d_3ab^2 + d_2a^2d_{14}d_9 - 3cos(\theta)ad_{11}b^2 - 3cos(\theta)ab^2d_{15} - 3cos(\theta)d_8a^2b - 3cos(\theta)ad_7b^2 + 2sin(\theta)d_4abd_{16} + sin(\theta)d_3d_8bd_{16} + 3d_1a^2d_{10}b + d_1a^2d_{10}d_{14} + sin(\theta)d_4d_8d_{11}b + d_1d_6d_{10}a^2 + d_1d_6d_{14}a^2 + 2d_1abd_{14}d_9 - d_1d_5d_9a^2 - sin(\theta)d_8a^2d_{15} - sin(\theta)a^2d_{12}d_{15}$$

Industrial Lubrication and Tribology
Volume 70 · Number 8 · 2018 · 1457–1475

$$
\begin{aligned}
&-2\cos(\theta)d_4ad_{11}b-2\cos(\theta)d_4abd_{15}-2\cos(\theta)d_4d_7ab\\
&-\cos(\theta)d_4d_7d_{11}b-\cos(\theta)d_4d_7bd_{15}-\cos(\theta)d_4bd_{11}d_{15}\\
&+\cos(\theta)d_4bd_{12}d_{16}-\sin(\theta)d_3d_{16}a^2+\cos(\theta)d_4d_8bd_{16}\\
&+2d_1ad_5d_{10}b-\sin(\theta)ad_7d_{16}d_{11}-\sin(\theta)ad_7d_{12}d_{15}\\
&+\cos(\theta)d_4d_8d_{12}b+\sin(\theta)d_8b^2d_{15}+3\sin(\theta)d_4ab^2\\
&-\sin(\theta)ad_8d_{11}d_{15}+\sin(\theta)ad_8d_{12}d_{16}+3d_1d_6a^2b\\
&+3d_1ad_5b^2-\sin(\theta)d_3ad_{16}d_{11}-\sin(\theta)d_3ad_{12}d_{15}\\
&-\sin(\theta)d_3d_7d_{16}a-2d_2ad_6bd_{14}-\sin(\theta)d_3d_7d_{12}a\\
&-\sin(\theta)d_3d_7d_{16}d_{11}-\sin(\theta)d_3d_8d_{11}a-\sin(\theta)d_3d_7d_{12}d_{15}\\
&-\sin(\theta)d_3d_8ad_{15}-2d_2ad_6d_{10}b+\sin(\theta)d_3d_8d_{12}d_{16}\\
&-\sin(\theta)d_3d_8d_{11}d_{15}-\sin(\theta)d_4ad_{11}d_{15}+\sin(\theta)d_4ad_{12}d_{16}\\
&-\sin(\theta)d_4d_7d_{11}a-\sin(\theta)d_4d_7ad_{15}-\sin(\theta)d_4d_7d_{11}d_{15}\\
&+\sin(\theta)d_4d_7d_{12}d_{16}+\sin(\theta)d_4d_8d_{16}a+\sin(\theta)d_4d_8d_{16}d_{11}\\
&+\sin(\theta)d_4d_8d_{12}a+\sin(\theta)d_4d_8d_{12}d_{15}-2\sin(\theta)ad_7d_{11}b\\
&-2\sin(\theta)ad_7bd_{15}-\sin(\theta)d_3d_{12}a^2-2\sin(\theta)abd_{11}d_{15}\\
&+2\sin(\theta)abd_{12}d_{16}+2d_2ad_5d_9b+2\sin(\theta)ad_8bd_{16}\\
&+\sin(\theta)d_4d_7b^2=0
\end{aligned}
\tag{B3}
$$

$$
\begin{aligned}
&-d_1a^2d_{10}d_{13}-d_1d_{14}a^3+\cos(\theta)d_{12}a^3-d_1d_{10}a^3-d_1d_6d_9a^2\\
&-d_2d_9a^3-d_1d_6a^3-d_2a^3d_{13}-\cos(\theta)d_3b^3+\cos(\theta)d_3d_7d_{12}d_{15}\\
&+\cos(\theta)d_3d_8ad_{15}+\cos(\theta)d_3d_8d_{11}a-3\sin(\theta)ab^2d_{15}\\
&-3\sin(\theta)d_8a^2b-3\sin(\theta)ad_7b^2-3\sin(\theta)ad_{11}b^2\\
&+\cos(\theta)d_3d_7d_{16}a+\cos(\theta)d_3d_7d_{16}d_{11}+\cos(\theta)d_3d_7d_{12}a\\
&-\cos(\theta)d_4d_7d_{12}b-\cos(\theta)d_4bd_{16}d_{11}-\sin(\theta)d_3d_8d_{16}a\\
&+\sin(\theta)d_7d_{11}a^2-\sin(\theta)d_8d_{16}a^2-\sin(\theta)d_8d_{12}a^2+d_1b^2d_{10}d_{13}\\
&+d_1d_6d_9b^2+d_1d_6b^2d_{13}+\sin(\theta)d_{12}b^3-\cos(\theta)d_4d_7b^2\\
&-\cos(\theta)d_4d_{11}b^2-\cos(\theta)d_4b^2d_{15}+\sin(\theta)a^2d_{11}d_{15}\\
&-\sin(\theta)d_3d_7d_{12}d_{16}-\sin(\theta)d_3d_8d_{16}d_{11}+3d_2d_6a^2b\\
&+3d_2a^2bd_{14}+3d_2a^2d_{10}b-\sin(\theta)d_3d_8d_{12}a\\
&-\sin(\theta)d_3d_8d_{12}d_{15}-\sin(\theta)d_4ad_{12}d_{15}-\sin(\theta)d_4ad_{16}d_{11}\\
&-\sin(\theta)d_4d_7d_{16}a-\cos(\theta)d_4d_8d_{16}a-\cos(\theta)d_4d_8d_{16}d_{11}\\
&-\cos(\theta)d_4d_8d_{12}a-\sin(\theta)d_4d_7d_{16}d_{11}-\sin(\theta)d_4d_7d_{12}a\\
&-\sin(\theta)d_4d_7d_{12}d_{15}-\sin(\theta)d_4d_8ad_{15}-\sin(\theta)d_4d_8d_{11}a\\
&+\sin(\theta)d_4d_8d_{12}d_{16}-\sin(\theta)d_4d_8d_{11}d_{15}+d_2d_6d_{14}a^2\\
&-2\sin(\theta)ad_7bd_{16}-2\sin(\theta)abd_{16}d_{11}-2\sin(\theta)ad_7d_{12}b\\
&-2\sin(\theta)abd_{12}d_{15}-2\sin(\theta)ad_8d_{11}b-2\sin(\theta)ad_8bd_{15}\\
&+\sin(\theta)d_3d_{11}a^2+\sin(\theta)d_3d_7a^2-\sin(\theta)d_4d_{16}a^2\\
&-3\sin(\theta)d_3ab^2-\sin(\theta)d_3d_7b^2-\sin(\theta)d_3d_{11}b^2\\
&+\cos(\theta)d_4a^2d_{15}+\cos(\theta)d_4d_{11}a^2+3\cos(\theta)a^2d_{11}b\\
&+3\cos(\theta)a^2bd_{15}+3\cos(\theta)d_7a^2b-3\cos(\theta)ab^2d_{16}\\
&-3\cos(\theta)ad_{12}b^2-\cos(\theta)b^2d_{16}d_{11}-\cos(\theta)b^2d_{12}d_{15}\\
&-\cos(\theta)d_8d_{11}b^2-\cos(\theta)d_8b^2d_{15}-3\cos(\theta)d_4ab^2\\
&-2\sin(\theta)d_3abd_{16}-2\sin(\theta)d_3ad_{12}b-\sin(\theta)d_3d_7bd_{16}\\
&-\sin(\theta)d_3d_7d_{12}b-\sin(\theta)d_3bd_{16}d_{11}-\sin(\theta)d_3bd_{12}d_{15}\\
&-2\sin(\theta)d_3d_8ab+3d_2ad_5b^2+3d_2ad_9b^2-\sin(\theta)d_3d_8d_{11}b\\
&-\sin(\theta)d_3d_8bd_{15}-\sin(\theta)bd_7d_{12}d_{15}-\sin(\theta)bd_7d_{16}d_{11}\\
&-\cos(\theta)d_4d_8bd_{15}-\cos(\theta)d_4d_8d_{11}b-\sin(\theta)bd_8d_{11}d_{15}\\
&+\sin(\theta)bd_8d_{12}d_{16}-2\sin(\theta)d_4abd_{15}-2\sin(\theta)d_4ad_{11}b\\
&-2\sin(\theta)d_4d_7ab-\sin(\theta)d_4d_7d_{11}b-\sin(\theta)d_4d_7bd_{15}\\
&+\cos(\theta)d_4d_7d_{11}d_{15}-\cos(\theta)d_4d_7d_{12}d_{16}+\sin(\theta)d_4bd_{12}d_{16}\\
&-\sin(\theta)d_4bd_{11}d_{15}+\sin(\theta)d_8d_{12}b^2-3\sin(\theta)d_4a^2b\\
&+\sin(\theta)d_4d_8bd_{16}+\sin(\theta)d_3ad_{11}d_{15}-d_1a^2d_{14}d_9\\
&-2\cos(\theta)d_4ad_{12}b-\cos(\theta)d_4d_7bd_{16}-d_2d_6b^2d_{14}\\
&-d_2d_6d_{10}b^2+\sin(\theta)d_4d_8d_{12}b-2d_1ad_5d_9b-2d_1ad_5bd_{13}\\
&-2d_1abd_9d_{13}+2d_1abd_{10}d_{14}+2d_1ad_6bd_{14}+2d_1ad_6d_{10}b\\
&-d_1bd_5d_9d_{13}+d_1bd_5d_{10}d_{14}+d_1bd_6d_{14}d_9+d_1bd_6d_{10}d_{13}\\
&+d_1d_5d_{10}b^2+d_1b^2d_{14}d_9-d_2d_6b^3+\cos(\theta)d_8a^3\\
&+\cos(\theta)d_{16}a^3+2d_2ad_5bd_{14}+2d_2ad_5d_{10}b+2d_2abd_{14}d_9\\
&+2d_2abd_{10}d_{13}-\cos(\theta)d_{11}b^3-\sin(\theta)d_3ad_{12}d_{16}\\
&+\sin(\theta)d_3d_7ad_{15}+2d_2ad_6d_9b+2d_2ad_6bd_{13}+d_2bd_5d_{14}d_9\\
&+d_2bd_5d_{10}d_{13}+d_2bd_6d_9d_{13}+\cos(\theta)d_3d_{12}a^2-d_2bd_6d_{10}d_{14}\\
&+d_1b^3d_{13}-\cos(\theta)b^3d_{15}-d_2b^3d_{14}+\sin(\theta)d_7a^3-d_2b^4\\
&+\sin(\theta)a^4+\sin(\theta)b^4+\sin(\theta)d_7a^2d_{15}-d_2d_5a^3\\
&+\cos(\theta)d_3d_8d_{11}d_{15}-\cos(\theta)d_3d_8d_{12}d_{16}+\cos(\theta)d_4ad_{11}d_{15}\\
&+2\cos(\theta)d_3d_7ab+2\cos(\theta)d_3ad_{11}b+2\cos(\theta)d_3abd_{15}\\
&-2\cos(\theta)ad_8bd_{16}-2\cos(\theta)ad_8d_{12}b+\cos(\theta)ad_8d_{11}d_{15}\\
&+\cos(\theta)ad_7d_{16}d_{11}+\cos(\theta)ad_7d_{12}d_{15}-\sin(\theta)a^2d_{12}d_{16}\\
&+3d_1ad_{10}b^2+3d_1ad_6b^2+\sin(\theta)ad_7d_{11}d_{15}\\
&-\sin(\theta)ad_7d_{12}d_{16}-\mathrm{d}_2a^2d_9d_{13}+d_2a^2d_{10}d_{14}\\
&-d_2d_5a^2d_{13}-d_2d_5d_9a^2+d_2d_6d_{10}a^2-ad_1d_5d_{14}d_9
\end{aligned}
$$

$$
\begin{aligned}
&- ad_1d_5d_{10}d_{13} - ad_1d_6d_9d_{13} + ad_1d_6d_{10}d_{14} - ad_2d_5d_9d_{13}\\
&+ ad_2d_5d_{10}d_{14} + ad_2d_6d_{14}d_9 + ad_2d_6d_{10}d_{13} + sin(\theta)d_3d_7d_{11}a\\
&+ sin(\theta)d_3d_7d_{11}d_{15} + 2cos(\theta)ad_7bd_{15} - 6sin(\theta)a^2b^2\\
&+ cos(\theta)d_4d_7ad_{15} + cos(\theta)d_4d_7d_{11}a + sin(\theta)d_8b^2d_{16}\\
&+ d_1d_5b^2d_{14} + cos(\theta)d_3d_{16}a^2 + cos(\theta)d_7d_{12}a^2\\
&+ cos(\theta)a^2d_{16}d_{11} + cos(\theta)d_8d_{11}a^2 + cos(\theta)d_3d_8a^2\\
&+ cos(\theta)d_4d_7a^2 + cos(\theta)d_8a^2d_{15} + cos(\theta)a^2d_{12}d_{15}\\
&+ cos(\theta)d_7d_{16}a^2 - cos(\theta)ad_8d_{12}d_{16} + cos(\theta)d_3ad_{16}d_{11}\\
&+ cos(\theta)d_3ad_{12}d_{15} - cos(\theta)bd_8d_{16}d_{11} + cos(\theta)bd_7d_{11}d_{15}\\
&- cos(\theta)bd_7d_{12}d_{16} + 2cos(\theta)abd_{11}d_{15} - 2cos(\theta)abd_{12}d_{16}\\
&+ sin(\theta)b^3d_{16} - 3cos(\theta)ad_8b^2 - cos(\theta)d_4ad_{12}d_{16}\\
&+ sin(\theta)d_4b^2d_{16} + 4cos(\theta)a^3b + cos(\theta)d_3d_7d_{11}b\\
&+ cos(\theta)d_3d_7bd_{15} - cos(\theta)d_4d_8d_{12}d_{15} + 2cos(\theta)ad_7d_{11}b\\
&- sin(\theta)d_4d_{12}a^2 - sin(\theta)d_4d_8a^2 - 3sin(\theta)a^2bd_{16}\\
&- sin(\theta)d_3b^2d_{15} - sin(\theta)d_7d_{11}b^2 - sin(\theta)d_7b^2d_{15}\\
&- sin(\theta)b^2d_{11}d_{15} + sin(\theta)b^2d_{12}d_{16} + sin(\theta)d_3a^2d_{15} + d_1d_9b^3\\
&- sin(\theta)ad_8d_{12}d_{15} + cos(\theta)d_3bd_{11}d_{15} - cos(\theta)d_3bd_{12}d_{16}\\
&+ d_2b^2d_9d_{13} - d_2b^2d_{10}d_{14} - 3d_1a^2bd_{13} - 3d_1d_5a^2b\\
&+ 3d_1ab^2d_{14} - cos(\theta)d_4bd_{12}d_{15} - 2cos(\theta)d_4d_8ab\\
&- cos(\theta)d_3d_8bd_{16} - cos(\theta)d_3d_8d_{12}b - cos(\theta)d_7d_{12}b^2\\
&+ 3cos(\theta)d_3a^2b - cos(\theta)d_3b^2d_{16} - cos(\theta)d_3d_{12}b^2\\
&- cos(\theta)d_3d_8b^2 - cos(\theta)d_7b^2d_{16} + 6d_2a^2b^2 + sin(\theta)d_4b^3\\
&+ sin(\theta)a^3d_{15} - d_1d_6a^2d_{13} - sin(\theta)ad_8d_{16}d_{11} + 3d_2ab^2d_{13}\\
&+ d_2d_5d_9b^2 + d_2d_5b^2d_{13} - 4d_1a^3b - cos(\theta)bd_8d_{12}d_{15}\\
&- 2cos(\theta)d_4abd_{16} + sin(\theta)d_4d_{12}b^2 + sin(\theta)d_4d_8b^2\\
&- cos(\theta)d_7b^3 + cos(\theta)d_4a^3 + sin(\theta)d_8b^3 - d_1d_5d_{10}a^2\\
&+ sin(\theta)d_{11}a^3 - d_1d_5d_{14}a^2 + d_1d_5b^3 - d_2d_{10}b^3 + 4d_1ab^3\\
&- 4cos(\theta)ab^3 + sin(\theta)d_3a^3 - 3d_1a^2d_9b - 3sin(\theta)a^2d_{12}b\\
&- d_2a^4 = 0
\end{aligned} \tag{B4}
$$

$$
\begin{aligned}
&- d_3d_8d_{11}d_{15} + d_3d_8d_{12}d_{16} + b^2d_{12}d_{15} + d_3b^2d_{16} + d_8b^2d_{15}\\
&+ d_3d_8b^2 + d_7d_{12}b^2 + d_3d_{12}b^2 + d_7b^2d_{16} + d_4d_{11}b^2\\
&+ d_4d_7b^2 + b^2d_{16}d_{11} - d_1d_{10}b^3 + d_8d_{11}b^2 + d_4b^2d_{15}\\
&- d_1d_6b^3 - d_1b^3d_{14} - d_2d_5b^3 - d_2d_9b^3 - d_2b^3d_{13} - d_2d_6d_9b^2\\
&- d_2d_6b^2d_{13} + d_3b^3 + d_7b^3 + d_{11}b^3 + b^3d_{15} - d_1b^4\\
&+ d_1bd_5d_{14}d_9 + d_1bd_5d_{10}d_{13} + d_1bd_6d_9d_{13} - d_1bd_6d_{10}d_{14}\\
&+ d_2bd_5d_9d_{13} - d_2bd_5d_{10}d_{14} - d_2bd_6d_{14}d_9 - d_2bd_6d_{10}d_{13}\\
&- d_3d_7d_{16}d_{11} + d_3d_8d_{12}b + d_4d_7d_{12}d_{16} + d_4d_8d_{16}d_{11}\\
&+ d_4d_8d_{12}d_{15} - d_3d_7d_{11}b - d_3d_7bd_{15} - d_3bd_{11}d_{15} + d_3bd_{12}d_{16}\\
&+ d_3d_8bd_{16} - d_4d_7d_{11}d_{15} + d_1d_5b^2d_{13} + d_1b^2d_9d_{13}\\
&- d_1b^2d_{10}d_{14} - d_1d_6b^2d_{14} - d_1d_6d_{10}b^2 - d_2d_5b^2d_{14}\\
&- d_2d_5d_{10}b^2 - d_2b^2d_{14}d_9 - d_2b^2d_{10}d_{13} - d_3d_7d_{12}d_{15}\\
&- bd_7d_{11}d_{15} + bd_7d_{12}d_{16} + bd_8d_{16}d_{11} + bd_8d_{12}d_{15} + d_4d_7bd_{16}\\
&+ d_4d_7d_{12}b + d_4bd_{16}d_{11} + d_4bd_{12}d_{15} + d_4d_8d_{11}b + d_4d_8bd_{15}\\
&+ d_1d_5d_9b^2 = 0
\end{aligned} \tag{B5}
$$

$$
\begin{aligned}
&d_1d_9b^3 - d_7b^2d_{15} - d_3d_7b^2 - d_7d_{11}b^2 + d_4b^2d_{16} - b^2d_{11}d_{15}\\
&+ d_4d_{12}b^2 - d_3d_{11}b^2 + b^2d_{12}d_{16} - d_3b^2d_{15} + d_8d_{12}b^2\\
&+ d_8b^2d_{16} + d_4d_8b^2 - d_2d_{10}b^3 - d_2d_6b^3 + d_1d_5b^3 + b^4\\
&- d_3d_8d_{11}b - d_3d_8bd_{15} - bd_7d_{16}d_{11} - bd_7d_{12}d_{15} - bd_8d_{11}d_{15}\\
&+ bd_8d_{12}d_{16} - d_4d_7d_{11}b - d_4d_7bd_{15} - d_4bd_{11}d_{15} + d_4bd_{12}d_{16}\\
&+ d_4d_8bd_{16} + d_4d_8d_{12}b + b^3d_{16} + d_{12}b^3 + d_8b^3 + d_4b^3\\
&+ d_1b^3d_{13} + d_1d_5b^2d_{14} + d_1d_5d_{10}b^2 + d_1b^2d_{14}d_9 + d_1b^2d_{10}d_{13}\\
&+ d_1d_6d_9b^2 + d_1d_6b^2d_{13} + d_2d_5d_9b^2 + d_2d_5b^2d_{13} + d_2b^2d_9d_{13}\\
&- d_2b^2d_{10}d_{14} - d_2d_6b^2d_{14} + d_2bd_6d_9d_{13} - d_1bd_5d_9d_{13}\\
&+ d_1bd_5d_{10}d_{14} + d_1bd_6d_{14}d_9 + d_1bd_6d_{10}d_{13} + d_2bd_5d_{14}d_9\\
&+ d_2bd_5d_{10}d_{13} - d_2d_6d_{10}b^2 - d_2b^3d_{14} - d_2bd_6d_{10}d_{14} - d_2b^4\\
&+ d_3d_7d_{11}d_{15} - d_3d_7d_{12}d_{16} - d_3d_8d_{16}d_{11} - d_3d_8d_{12}d_{15}\\
&- d_4d_7d_{16}d_{11} - d_4d_7d_{12}d_{15} - d_4d_8d_{11}d_{15} + d_4d_8d_{12}d_{16}\\
&- d_3d_7bd_{16} - d_3d_7d_{12}b - d_3bd_{16}d_{11} - d_3bd_{12}d_{15} = 0
\end{aligned} \tag{B6}
$$

$$
\begin{aligned}
&sin(\theta)d_4b^2d_{15} + d_2d_5d_{14}a^2 + d_1d_5d_9b^2 + 3d_2a^2d_9b\\
&- d_1bd_6d_{10}d_{14} + d_2a^2d_{10}d_{13} + d_1bd_5d_{14}d_9 - 3d_1ad_9b^2\\
&- 3d_1ad_5b^2 - d_2d_{14}a^3 + d_2bd_5d_9d_{13} + 3d_1d_6a^2b + cos(\theta)d_7a^3\\
&+ cos(\theta)a^3d_{15} - cos(\theta)d_{12}b^3 - cos(\theta)d_4b^3 - 4sin(\theta)ab^3\\
&+ sin(\theta)d_{16}a^3 + 4sin(\theta)a^3b + cos(\theta)d_8d_{16}a^2 + cos(\theta)d_8d_{12}a^2\\
&- cos(\theta)d_3a^2d_{15} - cos(\theta)d_3d_{11}a^2 - cos(\theta)d_3d_7a^2\\
&+ cos(\theta)d_4d_{16}a^2 + cos(\theta)d_4d_{12}a^2 + cos(\theta)d_4d_8a^2\\
&+ 3cos(\theta)a^2bd_{16} + 3cos(\theta)a^2d_{12}b + d_2d_5d_{10}a^2 - d_1d_6d_{10}b^2
\end{aligned}
$$

$$+ d_2d_6a^2d_{13} + d_2d_6d_9a^2 - cos(\theta)a^2d_{11}d_{15} + cos(\theta)a^2d_{12}d_{16}$$
$$- cos(\theta)d_7a^2d_{15} - cos(\theta)d_7d_{11}a^2 + ad_1d_5d_9d_{13} - ad_1d_5d_{10}d_{14}$$
$$- ad_1d_6d_{14}d_9 - ad_1d_6d_{10}d_{13} - ad_2d_5d_{14}d_9 - ad_2d_5d_{10}d_{13}$$
$$- ad_2d_6d_9d_{13} + ad_2d_6d_{10}d_{14} - d_2d_6a^3 + d_1a^3d_{13} + d_1d_9a^3$$
$$+ 6cos(\theta)a^2b^2 + d_1d_5a^3 - cos(\theta)d_8b^3 - d_2d_{10}a^3$$
$$+ cos(\theta)d_{11}a^3 + cos(\theta)d_3a^3 + sin(\theta)d_{12}a^3 + 2d_2ad_6bd_{14}$$
$$- 3d_1ab^2d_{13} - d_1a^4 - cos(\theta)b^3d_{16} + sin(\theta)d_{11}b^3 + sin(\theta)d_4a^3$$
$$+ sin(\theta)d_8a^3 + sin(\theta)d_3b^3 + sin(\theta)d_7b^3 - d_1b^3d_{14}$$
$$+ sin(\theta)b^3d_{15} + 6d_1a^2b^2 - d_1d_{10}b^3 - d_1d_6b^3 - 4d_2a^3b$$
$$+ 4d_2ab^3 - d_2bd_5d_{10}d_{14} - d_2b^2d_{10}d_{13} - d_2d_9b^3 - d_2d_5b^3$$
$$- d_2b^3d_{13} + cos(\theta)ad_7d_{11}d_{15} + 3d_2ad_{10}b^2 - cos(\theta)ad_7d_{12}d_{16}$$
$$- cos(\theta)ad_8d_{16}d_{11} - cos(\theta)ad_8d_{12}d_{15} + cos(\theta)d_3ad_{11}d_{15}$$
$$- cos(\theta)d_3ad_{12}d_{16} + cos(\theta)d_3d_7ad_{15} + cos(\theta)d_3d_7d_{11}a$$
$$- cos(\theta)d_3d_7d_{11}d_{15} + cos(\theta)d_3d_7d_{12}d_{16} - cos(\theta)d_3d_8d_{16}a$$
$$+ cos(\theta)d_3d_8d_{16}d_{11} - cos(\theta)d_3d_8d_{12}a - cos(\theta)d_4d_7d_{12}a$$
$$+ cos(\theta)d_3d_8d_{12}d_{15} - cos(\theta)d_4ad_{16}d_{11} - cos(\theta)d_4ad_{12}d_{15}$$
$$- cos(\theta)d_4d_7d_{16}a + cos(\theta)d_4d_7d_{16}d_{11} + cos(\theta)d_4d_7d_{12}d_{15}$$
$$- cos(\theta)d_4d_8ad_{15} - cos(\theta)d_4d_8d_{11}a + cos(\theta)d_4d_8d_{11}d_{15}$$
$$- cos(\theta)d_4d_8d_{12}d_{16} - 2cos(\theta)ad_7bd_{16} - 2cos(\theta)ad_7d_{12}b$$
$$- 2cos(\theta)abd_{16}d_{11} - 2cos(\theta)abd_{12}d_{15} - 2cos(\theta)ad_8d_{11}b$$
$$- 2cos(\theta)ad_8bd_{15} - 2cos(\theta)d_3abd_{16} - 2cos(\theta)d_3ad_{12}b$$
$$+ cos(\theta)d_3d_7bd_{16} + cos(\theta)d_3d_7d_{12}b + cos(\theta)d_3bd_{16}d_{11}$$
$$+ cos(\theta)d_3bd_{12}d_{15} - 2cos(\theta)d_3d_8ab + cos(\theta)d_3d_8d_{11}b$$
$$+ cos(\theta)d_3d_8bd_{15} + cos(\theta)bd_7d_{16}d_{11} + cos(\theta)bd_7d_{12}d_{15}$$
$$+ cos(\theta)bd_8d_{11}d_{15} - 2cos(\theta)d_4d_7ab - cos(\theta)bd_8d_{12}d_{16}$$
$$- 2cos(\theta)d_4ad_{11}b - 2cos(\theta)d_4abd_{15} + cos(\theta)d_4d_7d_{11}b$$
$$+ cos(\theta)d_4d_7bd_{15} + cos(\theta)d_4bd_{11}d_{15} - cos(\Theta)d_4bd_{12}d_{16}$$
$$- cos(\Theta)d_4d_8bd_{16} - cos(\theta)d_4d_8d_{12}b + sin(\theta)ad_7d_{16}d_{11}$$
$$+ sin(\theta)ad_7d_{12}d_{15} - d_2d_5b^2d_{14} + sin(\theta)ad_8d_{11}d_{15}$$
$$- sin(\theta)ad_8d_{12}d_{16} + sin(\theta)d_3ad_{16}d_{11} - d_1d_6b_2d_{14}$$
$$+ sin(\theta)d_3ad_{12}d_{15} + sin(\theta)d_3d_7d_{16}a - sin(\theta)d_3d_7d_{16}d_{11}$$
$$+ sin(\theta)d_3d_8ad_{15} + sin(\theta)d_3d_7d_{12}a - sin(\theta)d_3d_7d_{12}d_{15}$$
$$+ sin(\theta)d_3d_8d_{11}a - sin(\theta)d_3d_8d_{11}d_{15} + d_2a^2d_{14}d_9$$
$$+ d_1d_6d_{14}a^2 + d_1d_6d_{10}a^2 + sin(\theta)d_4d_{11}b^2 - d_1d_5d_9a^2$$
$$+ d_1a^2d_{10}d_{14} - d_2d_5d_{10}b^2 + d_1d_5b^2d_{13} - 2d_2ad_5bd_{13}$$
$$- d_2d_6b^2d_{13} + 3d_2ad_6b^2 + 3d_1a^2bd_{14} - d_2b^2d_{14}d_9$$
$$+ d_1bd_5d_{10}d_{13} - sin(\theta)d_4ad_{12}d_{16} + sin(\Theta)d_3d_8d_{12}d_{16}$$
$$+ sin(\Theta)d_4ad_{11}d_{15} - d_1a^2d_9d_{13} + sin(\theta)d_4d_7ad_{15}$$
$$+ sin(\theta)d_4d_7d_{11}a - d_2bd_6d_{10}d_{13} - sin(\theta)d_4d_7d_{11}d_{15}$$
$$+ sin(\theta)d_4d_7d_{12}d_{16} - sin(\theta)d_4d_8d_{16}a + sin(\theta)d_4d_8d_{16}d_{11}$$
$$- sin(\theta)d_4d_8d_{12}a + 2sin(\theta)ad_7d_{11}b - 2d_2abd_9d_{13}$$
$$+ 2d_2ad_6d_{10}b + sin(\theta)d_4d_8d_{12}d_{15} + 2sin(\theta)ad_7bd_{15}$$
$$+ 2sin(\theta)abd_{11}d_{15} - 2sin(\theta)abd_{12}d_{16} - 2sin(\theta)ad_8bd_{16}$$
$$+ 2sin(\theta)d_3abd_{15} - 2sin(\theta)ad_8d_{12}b + 2sin(\theta)d_3ad_{11}b$$
$$+ 2sin(\theta)d_3d_7ab - sin(\theta)d_3d_7d_{11}b - sin(\theta)d_3d_7bd_{15}$$
$$- sin(\theta)d_3bd_{11}d_{15} + sin(\theta)d_3bd_{12}d_{16} + sin(\theta)d_3d_8bd_{16}$$
$$+ sin(\theta)d_3d_8d_{12}b - sin(\theta)bd_7d_{11}d_{15} + sin(\theta)bd_8d_{16}d_{11}$$
$$+ sin(\theta)bd_7d_{12}d_{16} + sin(\theta)bd_8d_{12}d_{15} - 2sin(\theta)d_4abd_{16}$$
$$- cos(\theta)b^4 - 2sin(\theta)d_4ad_{12}b + sin(\theta)d_4d_7bd_{16}$$
$$+ sin(\theta)d_4d_7d_{12}b + sin(\theta)d_4bd_{16}d_{11} + sin(\theta)d_4bd_{12}d_{15}$$
$$- 2sin(\theta)d_4d_8ab + sin(\theta)d_4d_8d_{11}b + sin(\theta)d_4d_8bd_{15}$$
$$- 2d_1ad_5bd_{14} - 2d_1ad_5d_{10}b - 2d_1abd_{14}d_9 - 2d_1abd_{10}d_{13}$$
$$- cos(\theta)a^4 - 2d_1ad_{16}d_9b - 2d_1ad_6bd_{13} + d_1bd_6d_9d_{13}$$
$$+ cos(\theta)d_3d_{11}b^2 + cos(\theta)d_3d_7b^2 + 3cos(\theta)d_8a^2b$$
$$- 3cos(\theta)ad_7b^2 - 3cos(\theta)ad_{11}b^2 - 3cos(\theta)ab^2d_{15}$$
$$- 3cos(\theta)d_3ab^2 + cos(\theta)d_7d_{11}b^2 - d_1b^4 + cos(\theta)d_7b^2d_{15}$$
$$+ cos(\theta)b^2d_{11}d_{15} - cos(\theta)b^2d_{12}d_{16} - cos(\theta)d_8b^2d_{16}$$
$$+ cos(\theta)d_3b^2d_{15} - cos(\theta)d_8d_{12}b^2 + 3cos(\theta)d_4a^2b$$
$$- cos(\theta)d_4b^2d_{16} - cos(\theta)d_4d_{12}b^2 - cos(\theta)d_4d_8b^2$$
$$- 2d_2ad_5d_9b + 3d_2ab^2d_{14} - sin(\theta)d_3d_{12}a^2$$
$$- sin(\theta)d_3d_{16}a^2 - sin(\theta)d_7d_{12}a^2 - sin(-)a^2d_{16}d_{11}$$
$$+ 3d_2d_5a^2b - sin(\theta)d_8d_{11}a^2 - sin(\theta)d_3d_8a^2$$

$$
\begin{aligned}
&- sin(\theta)d_4d_7a^2 - sin(\theta)d_8a^2d_{15} - sin(\theta)a^2d_{12}d_{15} \\
&- sin(\theta)d_7d_{16}a^2 - in(\theta)d_4a^2d_{15} - sin(\theta)d_4d_{11}a^2 \\
&+ 3d_2a^2bd_{13} - 3sin(\theta)ab^2d_{16} - 3sin(\theta)a^2d_{11}b \\
&+ 2d_2abd_{10}d_{14} - 3sin(\theta)a^2bd_{15} - 3sin(\theta)d_7a^2b \\
&- d_1b^2d_{10}d_{14} - 3sin(\theta)ad_{12}b^2 - 3sin(\theta)ad_8b^2 \\
&- 3sin(\theta)d_3a^2b + sin(\theta)d_7b^2d_{16} + sin(\theta)d_3b^2d_{16} \\
&+ sin(\theta)d_3d_{12}b^2 + sin(\theta)d_3d_8b^2 + sin(\theta)d_7d_{12}b^2 \\
&+ sin(\theta)b^2d_{16}d_{11} + sin(\theta)b^2d_{12}d_{15} + 3d_1a^2d_{10}b - d_2bd_6d_{14}d_9 \\
&+ sin(\theta)d_8d_{11}b^2 + sin(\theta)d_8b^2d_{15} - 3sin(\theta)d_4ab^2 \\
&+ sin(\theta)d_4d_7b^2 - d_2d_6d_9b^2 - d_1d_5a^2d_{13} \\
&+ d_1b^2d_9d_{13} = 0
\end{aligned}
\tag{B7}
$$

$$
\begin{aligned}
&- d_1d_5d_{14}a^2 + 4d_1a^3b + cos(\theta)d_{12}a^3 + d_2d_9a^3 + d_1d_6a^3 \\
&+ d_2a^3d_{13} + sin(\theta)d_8b^3 + cos(\theta)d_3d_8b^2 - d_1d_5d_{10}a^2 \\
&- 6sin(\theta)a^2b^2 + d_1d_{14}a^3 - sin(\theta)d_4d_{16}a^2 - 3cos(\theta)ad_8b^2 \\
&- cos(\theta)d_4d_{11}a^2 + cos(\theta)d_3d_{12}b^2 - 3cos(\theta)d_7a^2b \\
&+ cos(\theta)d_4d_7b^2 - cos(\theta)a^2d_{12}d_{15} + cos(\theta)b^2d_{16}d_{11} \\
&+ 2sin(\theta)ad_8d_{11}b + sin(\theta)ad_8d_{16}d_{11} + sin(\theta)d_3d_7a^2 \\
&+ cos(\theta)d_3d_8d_{12}d_{16} - cos(\theta)d_3d_8d_{11}d_{15} + sin(\theta)d_3ad_{12}d_{16} \\
&- 3cos(\theta)d_4ab^2 - 3cos(\theta)ad_{12}b^2 - sin(\theta)ad_7d_{11}d_{15} \\
&+ sin(\theta)ad_7d_{12}d_{16} - d_2b^3d_{14} + 2cos(\theta)d_3d_7ab \\
&- 3sin(\theta)a^2bd_{16} - cos(\theta)a^2d_{16}d_{11} - cos(\theta)d_8d_{11}a^2 \\
&+ cos(\theta)d_4ad_{11}d_{15} + d_1d_9b^3 + sin(\theta)d_4b^3 + sin(\theta)b^3d_{16} \\
&+ 6d_2a^2b^2 + 4cos(\theta)a^3b - sin(\theta)a^2d_{12}d_{16} + d_2d_5a^3 \\
&- 2d_1ad_6bd_{14} - sin(\theta)d_3b^2d_{15} - 2cos(\theta)d_4ad_{12}b \\
&- cos(\theta)d_3d_{12}a^2 + sin(\theta)d_4d_8d_{11}a - sin(\theta)d_4d_8d_{11}d_{15} \\
&- d_2d_6b^2d_{14} - d_2d_6d_{10}b^2 + cos(\theta)d_4a^3 + cos(\theta)d_3b^3 \\
&- d_2a^4 + cos(\theta)d_7b^3 - sin(\theta)bd_7d_{12}d_{15} - sin(\theta)d_{11}a^3 \\
&+ cos(\theta)d_4d_7bd_{16} - d_2d_{10}b^3 + d_1b^3d_{13} - sin(\theta)d_7a^3 \\
&+ cos(\theta)d_4d_8d_{11}b + d_1d_{10}a^3 + sin(\theta)a^2d_{11}d_{15} - 2d_2ad_6d_9b \\
&+ sin(\theta)d_4d_8d_{12}b - cos(\theta)d_3d_{16}a^2 - cos(\theta)d_7d_{12}a^2 \\
&+ 2d_1ad_5d_9b - cos(\theta)d_8a^2d_{15} + cos(\theta)d_4bd_{12}d_{15} \\
&- d_2bd_6d_{10}d_{14} + cos(\theta)d_7d_{12}b^2 + sin(\theta)d_3a^2d_{15} + d_1d_5b^3 \\
&+ ad_1d_5d_{14}d_9 + d_2d_6d_{10}a^2 - sin(\theta)a^3d_{15} + cos(\theta)ad_7d_{16}d_{11} \\
&+ cos(\theta)ad_7d_{12}d_{15} + d_2d_5b^2d_{13} + d_2b^2d_9d_{13} - 3cos(\theta)a^2d_{11}b \\
&- d_2d_5a^2d_{13} - d_2d_5d_9a^2 + d_2d_6d_{14}a^2 - d_1d_6d_9a^2 \\
&-d_2a^2d_9d_{13} + d_2a^2d_{10}d_{14} - 2d_2ad_5bd_{14} - sin(\theta)d_3d_7bd_{16} \\
&+ ad_1d_5d_{10}d_{13} - sin(\theta)d_3ad_{11}d_{15} - sin(\theta)d_4d_7bd_{15} \\
&+ sin(\theta)d_4b^2d_{16} + ad_1d_6d_9d_{13} - 3sin(\theta)d_8a^2b \\
&- 3cos(\theta)a^2bd_{15} + sin(\theta)d_4d_{12}b^2 + sin(\theta)d_4d_8b^2 \\
&+ d_1bd_6d_{14}d_9 + d_1d_6d_9b^2 - d_1a^2d_{10}d_{13} - ad_1d_6d_{10}d_{14} \\
&+ d_1d_6b^2d_{13} - 4cos(\theta)ab^3 + ad_2d_5d_9d_{13} - d_1d_6a^2d_{13} \\
&+ sin(\theta)d_8d_{12}b^2 + 3sin(\theta)ad_7b^2 + cos(\theta)ad_8d_{11}d_{15} \\
&- cos(\theta)ad_8d_{12}d_{16} - sin(\theta)d_4d_{12}a^2 - ad_2d_5d_{10}d_{14} \\
&- sin(\theta)d_3a^3 + cos(\theta)d_4bd_{16}d_{11} - sin(\theta)b^2d_{11}d_{15} \\
&- 2d_2abd_{10}d_{13} - ad_2d_6d_{14}d_9 + cos(\theta)d_{16}a^3 - d_2d_6b^3 \\
&+ cos(\theta)d_8a^3 - 4d_1ab^3 + 3d_2a^2d_{10}b + 3d_2a^2bd_{14} \\
&- sin(\theta)d_8d_{12}a^2 - 2cos(\theta)abd_{12}d_{16} - 2cos(\theta)ad_8bd_{16} \\
&- sin(\theta)d_8d_{16}a^2 - 3cos(\theta)ab^2d_{16} + d_1bd_6d_{10}d_{13} \\
&+ cos(\theta)bd_7d_{12}d_{16} + cos(\theta)bd_8d_{16}d_{11} - 2d_2ad_6bd_{13} \\
&+ 2d_1abd_9d_{13} - sin(\theta)d_7b^2d_{15} - sin(\theta)bd_7d_{16}d_{11} \\
&- sin(\theta)d_3d_8bd_{15} + cos(\theta)d_4d_8bd_{15} - d_1bd_5d_9d_{13} \\
&+ cos(\theta)d_3d_8d_{11}a - 2d_1ad_6d_{10}b + 2d_1ad_5bd_{13} \\
&- cos(\theta)d_4d_7a^2 + d_2bd_5d_{10}d_{13} - cos(\theta)d_7d_{16}a^2 \\
&- cos(\theta)d_3d_8a^2 - cos(\theta)d_4a^2d_{15} + cos(\theta)d_3bd_{12}d_{16} \\
&+ cos(\theta)d_3d_8bd_{16} + cos(\theta)d_7b^2d_{16} + cos(\theta)d_3b^2d_{16} \\
&+ 3sin(\theta)ab^2d_{15} - 2d_2ad_5d_{10}b + sin(\theta)d_8b^2d_{16} \\
&+ sin(\theta)d_3d_{11}a^2 + cos(\theta)d_4d_7ad_{15} + cos(\theta)d_4d_7d_{11}a \\
&- cos(\theta)d_4d_7d_{11}d_{15} - 3d_1ad_{10}b^2 - 3d_1ad_6b^2 - 3d_1ab^2d_{14} \\
&+ sin(\theta)d_4bd_{12}d_{16} - sin(\theta)d_4bd_{11}d_{15} + cos(\theta)d_3ad_{16}d_{11} \\
&+ cos(\theta)d_{11}b^3 + 2sin(\theta)ad_7bd_{16} + 2sin(\theta)ad_7d_{12}b \\
&+ cos(\theta)d_3d_7d_{16}a - cos(\theta)d_3d_7d_{16}d_{11} - ad_2d_6d_{10}d_{13}
\end{aligned}
$$

$$
\begin{aligned}
&+ cos(\theta)d_8b^2d_{15} + sin(\theta)d_4d_8bd_{16} + cos(\theta)d_3d_8d_{12}b \\
&- cos(\theta)bd_7d_{11}d_{15} + cos(\theta)d_3ad_{12}d_{15} + d_1b^2d_{14}d_9 \\
&+ d_1b^2d_{10}d_{13} - cos(\theta)d_3d_7d_{11}b - cos(\theta)d_3d_7bd_{15} \\
&- cos(\theta)d_3bd_{11}d_{15} - sin(\theta)d_7d_{11}b^2 + 2sin(\theta)d_4ad_{11}b \\
&- d_2b^2d_{10}d_{14} + cos(\theta)bd_8d_{12}d_{15} - 2cos(\theta)d_4abd_{16} \\
&+ cos(\theta)d_4d_7d_{12}b + 3sin(\theta)ad_{11}b^2 - sin(\theta)d_3d_7d_{11}a \\
&+ sin(\theta)d_3d_7d_{11}d_{15} + cos(\theta)d_4d_{11}b^2 + 2sin(\theta)d_3d_8ab \\
&- sin(\theta)d_3bd_{12}d_{15} - 3sin(\theta)a^2d_{12}b + cos(\theta)b^2d_{12}d_{15} \\
&+ 2cos(\theta)ad_7d_{11}b + 2cos(\theta)ad_7bd_{15} + sin(\theta)d_4d_7d_{16}a \\
&- sin(\theta)d_4d_7d_{16}d_{11} + sin(\theta)d_4d_7d_{12}a + sin(\theta)bd_8d_{12}d_{16} \\
&-sin(\theta)bd_8d_{11}d_{15} + sin(\theta)ad_8d_{12}d_{15} + sin(\theta)d_4ad_{12}d_{15} \\
&- d_2b^4 + sin(\theta)a^4 + sin(\theta)b^4 - sin(\theta)d_3d_7d_{12}b \\
&+ cos(\theta)b^3d_{15} + 2sin(\theta)d_3ad_{12}b + sin(\theta)d_7a^2d_{15} \\
&+ sin(\theta)d_4d_8ad_{15} + d_1d_5d_{10}b^2 + cos(\theta)d_4d_7d_{12}d_{16} \\
&+ sin(\theta)d_7d_{11}a^2 - 2cos(\theta)d_4d_8ab - 3sin(\theta)d_4a^2b \\
&+ 2cos(\theta)d_3abd_{15} - sin(\theta)d_3d_8d_{12}d_{15} + 2sin(\theta)d_3abd_{16} \\
&+ 2sin(\theta)ad_8bd_{15} + 3d_2d_6a^2b - 3d_2ad_5b^2 + cos(\theta)d_4b^2d_{15} \\
&- cos(\theta)d_4d_8d_{12}a + cos(\theta)d_4d_8d_{12}d_{15} + 2sin(\theta)abd_{12}d_{15} \\
&+ 2sin(\theta)abd_{16}d_{11} - sin(\theta)d_4d_7d_{12}d_{15} - sin(\theta)d_3d_8d_{11}b \\
&+ sin(\theta)d_4ad_{16}d_{11} + d_2bd_5d_{14}d_9 + 2sin(\theta)d_4abd_{15} \\
&+ 3sin(\theta)d_3ab^2 + cos(\theta)d_3d_7d_{12}a - cos(\theta)d_3d_7d_{12}d_{15} \\
&+ cos(\theta)d_3d_8ad_{15} - d_1a^2d_{14}d_9 - sin(\theta)d_4d_8a^2 - 2d_1abd_{10}d_{14} \\
&+ 2cos(\theta)abd_{11}d_{15} - 3d_1a^2bd_{13} - 3d_1d_5a^2b \\
&- 2cos(\theta)ad_8d_{12}b + 2cos(\theta)d_3ad_{11}b - sin(\theta)d_3d_8d_{16}d_{11} \\
&+ sin(\theta)d_3d_8d_{12}a - sin(\theta)d_3d_7ad_{15} + sin(\theta)d_{12}b^3 \\
&- cos(\theta)d_4d_8d_{16}a + cos(\theta)d_4d_8d_{16}d_{11} - 3d_1a^2d_9b \\
&- 2d_2abd_{14}d_9 + 2sin(\theta)d_4d_7ab - sin(\theta)d_4d_7d_{11}b \\
&+ d_1bd_5d_{10}d_{14} + sin(\theta)d_4d_8d_{12}d_{16} - cos(\theta)d_4ad_{12}d_{16} \\
&+ d_2d_5d_9b^2 + d_2bd_6d_9d_{13} - 3d_2ad_9b^2 - 3d_2ab^2d_{13} \\
&- sin(\theta)d_3d_7d_{12}d_{16} + sin(\theta)d_3d_8d_{16}a - sin(\theta)d_3bd_{16}d_{11} \\
&+ cos(\theta)d_8d_{11}b^2 + sin(\theta)b^2d_{12}d_{16} - sin(\theta)d_3d_7b^2 \\
&- sin(\theta)d_3d_{11}b^2 - 3cos(\theta)d_3a^2b + d_1d_5b^2d_{14} = 0 \qquad \text{(B8)}
\end{aligned}
$$

$$
\begin{aligned}
&- ad_7d_{12}d_{16} + ad_7d_{11}d_{15} - ad_1d_5d_{10}d_{14} + d_{11}a^3 - d_4d_7d_{12}a \\
&- d_3d_7a^2 + d_4d_{16}a^2 + ad_1d_5d_9d_{13} + d_4d_{12}a^2 - ad_1d_6d_{10}d_{13} \\
&-ad_1d_6d_{14}d_9 - d_2d_{10}a^3 - d_3ad_{12}d_{16} + d_3ad_{11}d_{15} - ad_8d_{12}d_{15} \\
&- ad_8d_{16}d_{11} - ad_2d_5d_{10}d_{13} + a^3d_{15} + d_3d_7d_{12}d_{16} - d_3d_7d_{11}d_{15} \\
&+ d_3d_7d_{11}a + d_3d_7ad_{15} - ad_2d_5d_{14}d_9 + d_1d_5a^3 + a^2d_{12}d_{16} \\
&+ d_1d_9a^3 - d_3d_8d_{16}a - a^4 + d_3d_8d_{16}d_{11} - d_1a^4 - d_3d_8d_{12}a \\
&+ d_3a^3 + d_3d_8d_{12}d_{15} - ad_2d_6d_9d_{13} - d_3a^2d_{15} + d_4d_8a^2 \\
&- d_4d_7d_{16}a - d_4ad_{12}d_{15} - d_4ad_{16}d_{11} + d_1a^3d_{13} + d_4d_7d_{16}d_{11} \\
&+ d_7a^3 + ad_2d_6d_{10}d_{14} - a^2d_{11}d_{15} - d_3d_{11}a^2 + d_8d_{12}a^2 \\
&- d_2d_6a^3 + d_4d_7d_{12}d_{15} - d_4d_8ad_{15} - d_4d_8d_{11}a + d_4d_8d_{11}d_{15} \\
&- d_7d_{11}a^2 - d_4d_8d_{12}d_{16} - d_1a^2d_9d_{13} + d_8d_{16}a^2 + d_1a^2d_{10}d_{14} \\
&- d_1d_5a^2d_{13} - d_7a^2d_{15} - d_1d_5d_9a^2 + d_1d_6d_{14}a^2 - d_2d_{14}a^3 \\
&+ d_1d_6d_{10}a^2 + d_2a^2d_{14}d_9 + d_2a^2d_{10}d_{13} + d_2d_5d_{14}a^2 \\
&+ d_2d_5d_{10}a^2 + d_2d_6a^2d_{13} + d_2d_6d_9a^2=0 \qquad \text{(B9)}
\end{aligned}
$$

$$
\begin{aligned}
&- d_4d_8d_{12}a + d_4d_8d_{12}d_{15} - d_1a^2d_{14}d_9 - d_1a^2d_{10}d_{13} \\
&- d_1d_5d_{14}a^2 - d_7d_{12}a^2 - d_4d_8d_{16}a + d_4d_8d_{16}d_{11} - d_2d_5a^2d_{13} \\
&- d_1d_5d_{10}a^2 - d_1d_6a^2d_{13} - d_1d_6d_9a^2 - d_2a^2d_9d_{13} \\
&+ d_2a^2d_{10}d_{14} - d_3d_{16}a^2 - d_2d_5d_9a^2 + d_2d_6d_{14}a^2 + d_2d_6d_{10}a^2 \\
&+ ad_1d_5d_{14}d_9 + ad_1d_5d_{10}d_{13} + ad_1d_6d_9d_{13} - ad_1d_6d_{10}d_{14} \\
&+ d_{16}a^3 + d_{12}a^3 + d_8a^3 + d_4a^3 - d_2a^4 + ad_7d_{16}d_{11} \\
&+ ad_7d_{12}d_{15} + ad_8d_{11}d_{15} - ad_8d_{12}d_{16} + d_3ad_{16}d_{11} + d_3ad_{12}d_{15} \\
&+ d_3d_7d_{16}a - d_3d_7d_{16}d_{11} + d_3d_7d_{12}a - d_3d_7d_{12}d_{15} + d_3d_8ad_{15} \\
&+ d_3d_8d_{11}a - d_3d_8d_{11}d_{15} + d_3d_8d_{12}d_{16} + d_4ad_{11}d_{15} \\
&- d_4ad_{12}d_{16} + d_4d_7ad_{15} + d_4d_7d_{11}a - d_4d_7d_{11}d_{15} + d_4d_7d_{12}d_{16} \\
&- d_8d_{11}a^2 + ad_2d_5d_9d_{13} - ad_2d_5d_{10}d_{14} - ad_2d_6d_{14}d_9 \\
&- ad_2d_6d_{10}d_{13} + d_1d_{10}a^3 + d_1d_{14}a^3 - d_4a^2d_{15} + d_2d_5a^3 \\
&- d_4d_{11}a^2 - a^2d_{12}d_{15} - d_3d_8a^2 - a^2d_{16}d_{11} + d_1d_6a^3 \\
&- d_3d_{12}a^2 + d_2a^3d_{13} - d_8a^2d_{15} - d_7d_{16}a^2 - d_4d_7a^2 \\
&+ d_2d_9a^3 = 0 \qquad \text{(B10)}
\end{aligned}
$$

$$d_1d_6d_{10}a^2 + \cos(\theta)d_{11}a^3 - d_2d_6a^3 + d_1a^3d_{13} + \cos(\theta)d_7a^3$$
$$+ d_1d_5a^3 + d_1d_9a^3 - d_2d_{10}a^3 - d_1d_5a^2d_{13} + d_2a^2d_{14}d_9$$
$$- \cos(\theta)d_4d_7d_{12}a + \cos(\theta)d_4d_7d_{12}d_{15} - \cos(\theta)d_4d_8ad_{15}$$
$$- \cos(\theta)d_4d_8d_{11}a + \cos(\theta)d_4d_8d_{11}d_{15} + d_1a^2d_{10}d_{14}$$
$$+ d_1d_6d_{14}a^2 + 2\sin(\theta)d_3ad_{11}b - \cos(\theta)d_3a^2d_{15}$$
$$- 2\sin(\theta)ad_8d_{12}b - \sin(\theta)d_3d_8ad_{15} - \sin(\theta)d_3d_8d_{11}a$$
$$+ \sin(\theta)d_3d_8d_{11}d_{15} + 2\sin(\theta)d_3abd_{15} + 2\sin(\theta)d_3d_7ab$$
$$+ \cos(\theta)d_3d_7ad_{15} + \cos(\theta)d_3d_7d_{11}a - \cos(\theta)d_3d_7d_{11}d_{15}$$
$$+ \cos(\theta)d_3d_7d_{12}d_{16} - \cos(\theta)d_3d_8d_{16}a + \cos(\theta)d_3d_8d_{16}d_{11}$$
$$- \sin(\theta)d_3d_7d_{11}b - \sin(\theta)d_3d_7bd_{15} - \sin(\Theta)d_3bd_{11}d_{15}$$
$$+ \sin(\theta)d_3bd_{12}d_{16} + \sin(\theta)d_3d_8bd_{16} + \sin(\theta)d_3d_8d_{12}b$$
$$+ \cos(\theta)d_{12}b^3 - d_1a^2d_9d_{13} - \sin(\theta)bd_7d_{11}d_{15}$$
$$+ \sin(\theta)bd_7d_{12}d_{16} + \sin(\theta)bd_8d_{16}d_{11} + \sin(\theta)bd_8d_{12}d_{15}$$
$$- 2\sin(\theta)d_4abd_{16} - 2\sin(\theta)d_4ad_{12}b + \sin(\theta)d_4d_7bd_{16}$$
$$+ \sin(\theta)d_4d_7d_{12}b + \sin(\theta)d_4bd_{16}d_{11} + \sin(\theta)d_4bd_{12}d_{15}$$
$$- 2\sin(\theta)d_4d_8ab + \sin(\theta)d_4d_8d_{11}b + \sin(\theta)d_4d_8bd_{15}$$
$$+ 2d_1ad_5bd_{14} + 2d_1ad_5d_{10}b + 2d_1abd_{14}d_9 - \cos(\theta)a^4$$
$$+ 2d_1abd_{10}d_{13} + 2d_1ad_6d_9b + 2d_1ad_6bd_{13} - 3\cos(\theta)d_8a^2b$$
$$- 3\cos(\theta)ad_7b^2 - 3\cos(\theta)ad_{11}b^2 - 3\cos(\theta)ab^2d_{15}$$
$$- 3\cos(\theta)d_3ab^2 + \cos(\theta)d_3d_7b^2 + \cos(\theta)d_3d_{11}b^2$$
$$+ \cos(\theta)d_3b^2d_{15} + \cos(\theta)d_7d_{11}b^2 + \cos(\theta)d_7b^2d_{15}$$
$$+ \cos(\theta)b^2d_{11}d_{15} - \cos(\theta)b^2d_{12}d_{16} - \cos(\theta)d_8b^2d_{16}$$
$$- \cos(\theta)d_4b^2d_{16} - \cos(\theta)d_4d_{12}b^2 - \cos(\theta)bd_7d_{16}d_{11}$$
$$- \cos(\theta)bd_7d_{12}d_{15} - \cos(\theta)bd_8d_{11}d_{15} - \cos(\theta)d_8d_{12}b^2$$
$$- 3\cos(\theta)d_4a^2b - d_2d_{14}a^3 - \cos(\theta)d_4d_8b^2 + \sin(\theta)d_3d_{12}a^2$$
$$+ \sin(\theta)d_3d_{16}a^2 + \sin(\theta)d_7d_{12}a^2 + \sin(\theta)a^2d_{16}d_{11}$$
$$+ \sin(\theta)d_8d_{11}a^2 + \sin(\theta)d_3d_8a^2 + \sin(\theta)d_4d_7a^2$$
$$+ \sin(\theta)d_8a^2d_{15} + \sin(\theta)a^2d_{12}d_{15} + \sin(\theta)d_7d_{16}a^2$$
$$+ \sin(\theta)d_4a^2d_{15} + \sin(\theta)d_4d_{11}a^2 - 3\sin(\theta)a^2d_{11}b$$
$$+ d_2a^2d_{10}d_{13} + d_2d_5d_{14}a^2 + d_2d_5d_{10}a^2 + d_2d_6a^2d_{13}$$
$$+ d_2d_6d_9a^2 - \cos(\theta)a^2d_{11}d_{15} + \cos(\theta)a^2d_{12}d_{16}$$
$$- \cos(\theta)d_7a^2d_{15} - \cos(\theta)d_7d_{11}a^2 + \cos(\theta)d_8d_{16}a^2$$
$$+ \cos(\theta)d_8d_{12}a^2 - \cos(\theta)d_3d_{11}a^2 + ad_1d_5d_9d_{13}$$
$$- 3\sin(\theta)a^2bd_{15} - 3\sin(\theta)d_7a^2b + 3\sin(\theta)ab^2d_{16}$$
$$+ 3\sin(\theta)ad_{12}b^2 - \sin(\theta)d_3d_{12}b^2 - ad_1d_5d_{10}d_{14} - ad_1d_6d_{14}d_9$$
$$- ad_1d_6d_{10}d_{13} + 3\sin(\theta)ad_8b^2 - 3\sin(\theta)d_3a^2b$$
$$- \sin(\theta)d_3b^2d_{16} - \sin(\theta)b^2d_{16}d_{11} - \sin(\theta)d_3d_8b^2$$
$$- \sin(\theta)d_7b^2d_{16} - \sin(\theta)d_7d_{12}b^2 - \sin(\theta)b^2d_{12}d_{15}$$
$$- \sin(\theta)d_8d_{11}b^2 - \sin(\theta)d_8b^2d_{15} + 3\sin(\theta)d_4ab^2$$
$$- \sin(\theta)d_4d_7b^2 - \sin(\theta)d_4d_{11}b^2 + \cos(\theta)d_3a^3 + d_2d_5b^3$$
$$- 3d_1a^2d_{10}b - 3d_1d_6a^2b - \sin(\theta)d_4b^2d_{15} - 3d_1a^2bd_{14}$$
$$- 3d_1ad_5b^2 - 3d_1ad_9b^2 - 3d_1ab^2d_{13} + d_1d_5d_9b^2$$
$$+ d_1d_5b^2d_{13} + d_1b^2d_9d_{13} - d_1b^2d_{10}d_{14} - d_1d_6b^2d_{14}$$
$$- 3d_2a^2bd_{13} - d_1d_6d_{10}b^2 - 3d_2a^2d_9b - 3d_2d_5a^2b$$
$$+ 3d_2ab^2d_{14} - d_2d_5b^2d_{14} + 3d_2ad_{10}b^2 + 3d_2ad_6b^2$$
$$- \sin(\theta)d_4d_8d_{12}d_{15} - d_2d_5d_{10}b^2 - d_2b^2d_{14}d_9 - d_2b^2d_{10}d_{13}$$
$$- d_2d_6d_9b^2 - d_2d_6b^2d_{13} + d_2d_9b^3 + d_2b^3d_{13}$$
$$+ \cos(\theta)d_4d_8d_{12}b - \sin(\theta)ad_7d_{16}d_{11} - \cos(\theta)b^4 - d_1b^4$$
$$- d_1bd_5d_{14}d_9 + \cos(\theta)d_8b^3 - d_1bd_5d_{10}d_{13} - d_1bd_6d_9d_{13}$$
$$+ d_1bd_6d_{10}d_{14} + 2d_2ad_5bd_{13} + 2d_2ad_5d_9b + 2d_2abd_9d_{13}$$
$$-2d_2abd_{10}d_{14} - 2d_2ad_6bd_{14} - 2d_2ad_6d_{10}b - \cos(\theta)d_3d_8d_{12}a$$
$$+ \cos(\theta)d_3d_8d_{12}d_{15} - \cos(\theta)d_4ad_{16}d_{11} - \cos(\theta)d_4ad_{12}d_{15}$$
$$- \cos(\theta)d_4d_7d_{16}a + \cos(\theta)d_4d_7d_{16}d_{11} - d_2bd_5d_9d_{13}$$
$$+ d_2bd_5d_{10}d_{14} + d_2bd_6d_{14}d_9 + d_2bd_6d_{10}d_{13} - \sin(\theta)d_3ad_{12}d_{15}$$
$$- 3\cos(\theta)a^2d_{12}b - \sin(\theta)ad_7d_{12}d_{15} - \sin(\theta)d_3d_7d_{12}a$$
$$+ \sin(\theta)d_3d_7d_{12}d_{15} - \cos(\theta)d_3ad_{12}d_{16} + \cos(\theta)ad_7d_{11}d_{15}$$
$$- \cos(\theta)ad_7d_{12}d_{16} - \cos(\theta)ad_8d_{16}d_{11} - \cos(\theta)ad_8d_{12}d_{15}$$
$$+ \cos(\theta)d_3ad_{11}d_{15} + \cos(\theta)bd_8d_{12}d_{16} + 2\cos(\theta)d_4ad_{11}b$$
$$+ 2\cos(\theta)d_4abd_{15} - ad_2d_5d_{14}d_9 - ad_2d_5d_{10}d_{13} - ad_2d_6d_9d_{13}$$
$$+ ad_2d_6d_{10}d_{14} + 2\cos(\theta)d_3ad_{12}b - \cos(\theta)d_3d_7bd_{16}$$
$$- \cos(\theta)d_3d_7d_{12}b - \cos(\theta)d_3bd_{16}d_{11} + \cos(\theta)a^3d_{15}$$
$$+ 2\sin(\theta)abd_{11}d_{15} + 2\sin(\theta)ad_7d_{11}b + 2\sin(\theta)ad_7bd_{15}$$
$$+ \sin(\theta)d_3d_7d_{16}d_{11} - \sin(\theta)d_3d_7d_{16}a - d_1a^4 + 6\cos(\theta)a^2b^2$$
$$+ \cos(\theta)b^3d_{16} + \sin(\theta)d_4d_8d_{16}a - \sin(\theta)d_4d_8d_{16}d_{11}$$
$$+ \sin(\theta)d_4d_7d_{11}d_{15} - \sin(\theta)d_4d_7d_{12}d_{16} - \sin(\theta)d_3d_8d_{12}d_{16}$$
$$- \sin(\theta)d_4ad_{11}d_{15} - 2\sin(\theta)abd_{12}d_{16} - 2\sin(\theta)ad_8bd_{16}$$
$$- \cos(\theta)d_4d_8d_{12}d_{16} + 2\cos(\theta)ad_7bd_{16} + 2\cos(\theta)ad_7d_{12}b$$
$$+ 2\cos(\theta)abd_{16}d_{11} - \cos(\theta)d_3bd_{12}d_{15} + 2\cos(\theta)d_3d_8ab$$
$$- \cos(\theta)d_3d_8bd_{15} - \cos(\theta)d_3d_8d_{11}b + \sin(\theta)d_7b^3 + \sin(\theta)d_3b^3$$
$$+ \sin(\theta)d_{11}b^3 + 6d_1a^2b^2 + d_1d_{10}b^3 + \sin(\theta)b^3d_{15} + d_1b^3d_{14}$$

$$
\begin{aligned}
&+ d_1d_6b^3 - 4d_2ab^3 + 4d_2a^3b + sin(\theta)d_4d_8d_{12}a \\
&- sin(\theta)d_4d_7d_{11}a + sin(\theta)d_4ad_{12}d_{16} - sin(\theta)(\theta)d_4d_7ad_{15} \\
&+ cos(\theta)d_4b^3 + 2cos(\theta)d_4d_7ab - cos(\theta)d_4d_7d_{11}b \\
&- cos(\theta)d_4d_7bd_{15} - 3cos(\theta)a^2bd_{16} - cos(\theta)d_3d_7a^2 \\
&+ cos(\theta)d_4d_{16}a^2 + cos(\theta)d_4d_{12}a^2 + cos(\theta)d_4d_8a^2 \\
&+ 2cos(\theta)abd_{12}d_{15} + 2cos(\theta)ad_8d_{11}b + 2cos(\theta)ad_8bd_{15} \\
&+ 2cos(\theta)d_3abd_{16} - 4sin(\theta)ab^3 - sin(\theta)d_8a^3 - sin(\theta)d_4a^3 \\
&- sin(\theta)ad_8d_{11}d_{15} + sin(\theta)ad_8d_{12}d_{16} - sin(\theta)d_3ad_{16}d_{11} \\
&- sin(\theta)d_{16}a^3 - d_1d_5d_9a^2 - sin(\theta)d_{12}a^3 - cos(\theta)d_4bd_{11}d_{15} \\
&+ cos(\theta)d_4bd_{12}d_{16} + cos(\theta)d_4d_8bd_{16} \\
&+ 4sin(\theta)a^3b = 0 \qquad \text{(B11)}
\end{aligned}
$$

$$
\begin{aligned}
&- cos(\theta)d_3d_8bd_{16} - d_1a^2d_{14}d_9 + 2d_1ad_6d_{10}b + cos(\theta)ad_7d_{16}d_{11} \\
&- d_1a^2d_{10}d_{13} - 3d_2ad_5b^2 + 2sin(\theta)abd_{16}d_{11} \\
&+ cos(\theta)d_3d_8d_{12}d_{16} - 2cos(\theta)d_3ad_{11}b + 2cos(\theta)abd_{12}d_{16} \\
&+ cos(\theta)ad_7d_{12}d_{15} + 2sin(\theta)d_3abd_{16} + 2sin(\theta)abd_{12}d_{15} \\
&+ 2d_1ad_6bd_{14} - d_1d_5d_{14}a^2 + 2sin(\theta)ad_7bd_{16} \\
&+ sin(\theta)d_4d_7d_{16}d_{11} + d_2bd_6d_{10}d_{14} - cos(\theta)d_3d_{12}a^2 - d_1b^3d_{13} \\
&- cos(\theta)d_{11}b^3 - cos(\theta)b^3d_{15} + sin(\theta)d_7a^3 - sin(\theta)d_4ad_{12}d_{15} \\
&- d_2d_6d_{10}b^2 - sin(\theta)d_4d_8d_{12}d_{16} + cos(\theta)d_3ad_{16}d_{11} \\
&- sin(\theta)bd_7d_{12}d_{15} - sin(\theta)bd_8d_{11}d_{15} + sin(\theta)d_4d_7d_{12}d_{15} \\
&+ sin(\theta)d_3d_8d_{12}d_{15} - cos(\theta)ad_8d_{12}d_{16} - sin(\theta)d_3d_7bd_{16} \\
&+ cos(\theta)ad_8d_{11}d_{15} + 2d_2ad_6d_9b + 2d_1abd_{10}d_{14} - d_2d_6b^2d_{14} \\
&- cos(\theta)d_3d_7d_{16}d_{11} - d_2b^2d_{10}d_{14} + d_2b^2d_9d_{13} \\
&- sin(\theta)d_4d_8ad_{15} + 2d_2ad_5bd_{14} + d_2b^3d_{14} + cos(\theta)d_3d_7d_{16}a \\
&- cos(\theta)bd_7d_{12}d_{16} + sin(\theta)d_3d_8d_{16}d_{11} + d_2d_5d_9b^2 \\
&- 2cos(\theta)ad_7d_{11}b + 2d_2abd_{14}d_9 + sin(\theta)d_3ad_{11}d_{15} \\
&-2d_1ad_5bd_{13} + d_2d_5b^2d_{13} + cos(\theta)d_3ad_{12}d_{15} - cos(\theta)d_4d_7bd_{16} \\
&- sin(\theta)bd_7d_{16}d_{11} + 2sin(\theta)d_4d_7ab + sin(\theta)d_4d_8bd_{16} \\
&+ d_1bd_5d_9d_{13} + sin(\theta)d_3d_7d_{11}a - d_1bd_6d_{10}d_{13} + 2d_2abd_{10}d_{13}
\end{aligned}
$$

$$
\begin{aligned}
&+ sin(\theta)ad_7d_{11}d_{15} + sin(\theta)d_4d_8d_{12}b - cos(\theta)d_3d_7d_{12}d_{15} \\
&- sin(\theta)d_4d_7bd_{15} - sin(\theta)d_4bd_{11}d_{15} - cos(\theta)bd_8d_{16}d_{11} \\
&-cos(\theta)d_4d_8d_{16}a + sin(\theta)d_{12}b^3 - 2d_1ad_5d_9b + 2cos(\theta)d_4ad_{12}b \\
&+ 2sin(\theta)d_4ad_{11}b - 3d_2ab^2d_{13} + 2sin(\theta)ad_7d_{12}b \\
&- cos(\theta)d_4d_8d_{11}b - 3d_2ad_9b^2 + 2cos(\theta)d_4d_8ab \\
&- sin(\theta)d_3d_8d_{12}a - sin(\theta)b^4 - d_2b^4 - sin(\theta)a^4 \\
&+ cos(\theta)d_4ad_{11}d_{15} - cos(\theta)d_4ad_{12}d_{16} - d_2bd_6d_9d_{13} \\
&+ 2sin(\theta)d_4abd_{15} + 2cos(\theta)ad_8d_{12}b - sin(\theta)d_3d_8bd_{15} \\
&- sin(\theta)ad_8d_{12}d_{15} + cos(\theta)d_4d_7d_{11}a - cos(\theta)d_4d_7d_{11}d_{15} \\
&+ cos(\theta)d_4d_7ad_{15} + cos(\theta)d_4d_8d_{16}d_{11} - cos(\theta)d_4bd_{16}d_{11} \\
&+ cos(\theta)d_3d_7d_{12}a - cos(\theta)bd_8d_{12}d_{15} + sin(\theta)d_4bd_{12}d_{16} \\
&+ cos(\theta)d_4d_8d_{12}d_{15} - cos(\theta)d_4d_8d_{12}a + cos(\theta)bd_7d_{11}d_{15} \\
&- cos(\theta)d_4bd_{12}d_{15} - sin(\theta)d_4d_7d_{11}b - cos(\theta)d_7b^3 + 6d_2a^2b^2 \\
&+ cos(\theta)d_{12}a^3 - cos(\theta)d_3b^3 + 6sin(\theta)a^2b^2 + d_1d_6a^3 + d_2d_9a^3 \\
&+ d_1d_{10}a^3 + d_2a^3d_{13} + d_1d_{14}a^3 + sin(\theta)d_4b^3 - d_1d_9b^3 \\
&+ d_2d_5a^3 + sin(\theta)b^3d_{16} - 4cos(\theta)a^3b + 2d_2ad_5d_{10}b \\
&- cos(\theta)d_3d_8d_{11}d_{15} - ad_2d_6d_{10}d_{13} - ad_1d_6d_{10}d_{14} + ad_2d_5d_9d_{13} \\
&- ad_2d_5d_{10}d_{14} - ad_2d_6d_{14}d_9 + ad_1d_5d_{14}d_9 + ad_1d_5d_{10}d_{13} \\
&+ ad_1d_6d_9d_{13} - d_1d_6a^2d_{13} - d_1d_6d_9a^2 - d_2a^2d_9d_{13} \\
&+ d_2a^2d_{10}d_{14} - d_2d_5a^2d_{13} - d_2d_5d_9a^2 + d_2d_6d_{14}a^2 \\
&+ d_2d_6d_{10}a^2 + sin(\theta)d_4d_8d_{11}d_{15} + cos(\theta)d_3d_8d_{11}a \\
&- sin(\theta)ad_8d_{16}d_{11} - 4d_1a^3b - d_1d_5d_{10}a^2 - d_1bd_6d_{14}d_9 \\
&+ 2cos(\theta)d_4abd_{16} - d_2a^4 - sin(\theta)d_4ad_{16}d_{11} + d_2d_{10}b^3 \\
&+ sin(\theta)d_{11}a^3 + cos(\theta)d_4a^3 + sin(\theta)a^3d_{15} - d_1d_5b^3 \\
&+ cos(\theta)d_8a^3 + sin(\theta)d_8b^3 + 4cos(\theta)ab^3 + sin(\theta)d_3a^3 + 4d_1ab^3 \\
&+ d_2d_6b^3 + cos(\theta)d_{16}a^3 - cos(\theta)a^2d_{16}d_{11} - cos(\theta)d_3d_{16}a^2 \\
&- cos(\theta)d_7d_{12}a^2 - cos(\theta)d_4d_8bd_{15} + cos(\theta)d_3d_7d_{11}b \\
&+ cos(\theta)d_4d_7d_{12}d_{16} + 2sin(\theta)d_3d_8ab - cos(\theta)d_8d_{11}a^2
\end{aligned}
$$

$$\begin{aligned}
&-cos(\theta)d_3d_8a^2-cos(\theta)d_4d_7a^2-cos(\theta)d_8a^2d_{15}\\
&-cos(\theta)a^2d_{12}d_{15}-cos(\theta)d_4a^2d_{15}-cos(\theta)d_4d_{11}a^2\\
&-cos(\theta)d_4d_7d_{12}b-cos(\theta)d_7d_{16}a^2-sin(\theta)d_3d_8d_{16}a\\
&-2cos(\theta)ad_7bd_{15}+3cos(\theta)a^2d_{11}b+3cos(\theta)a^2bd_{15}\\
&+3cos(\theta)d_7a^2b-3cos(\theta)ab^2d_{16}+2cos(\theta)ad_8bd_{16}\\
&-3cos(\theta)ad_{12}b^2-3cos(\theta)ad_8b^2+3cos(\theta)d_3a^2b\\
&+sin(\theta)d_3d_7ad_{15}-sin(\theta)d_3bd_{12}d_{15}+cos(\theta)b^2d_{16}d_{11}\\
&+cos(\theta)d_3b^2d_{16}+cos(\theta)d_3d_{12}b^2+cos(\theta)d_3d_8b^2\\
&+cos(\theta)d_7b^2d_{16}+cos(\theta)d_7d_{12}b^2-2cos(\theta)abd_{11}d_{15}\\
&+cos(\theta)b^2d_{12}d_{15}+cos(\theta)d_8d_{11}b^2+cos(\theta)d_8b^2d_{15}\\
&-3cos(\theta)d_4ab^2+cos(\theta)d_4d_7b^2+2d_2ad_6bd_{13}+cos(\theta)d_4d_{11}b^2\\
&+cos(\theta)d_4b^2d_{15}-sin(\theta)a^2d_{11}d_{15}+sin(\theta)a^2d_{12}d_{16}\\
&-sin(\theta)d_7a^2d_{15}+sin(\theta)d_3d_7d_{12}d_{16}+sin(\theta)d_8d_{16}a^2\\
&-2cos(\theta)d_3d_7ab+sin(\theta)d_8d_{12}a^2-sin(\theta)d_3a^2d_{15}\\
&-sin(\theta)d_3d_{11}a^2-sin(\theta)d_3d_7d_{12}b-sin(\theta)d_3bd_{16}d_{11}\\
&-sin(\theta)d_7d_{11}a^2-d_1bd_5d_{10}d_{14}+sin(\theta)d_4d_{16}a^2\\
&+sin(\theta)d_4d_{12}a^2+sin(\theta)d_4d_8a^2-sin(\theta)d_3d_7a^2\\
&-sin(\theta)d_3d_8d_{11}b-3sin(\theta)a^2bd_{16}-3sin(\theta)a^2d_{12}b\\
&-3sin(\theta)d_8a^2b-3sin(\theta)ad_7b^2+cos(\theta)d_3d_7bd_{15}\\
&-sin(\theta)ad_7d_{12}d_{16}+2sin(\theta)ad_8d_{11}b+sin(\theta)bd_8d_{12}d_{16}\\
&+sin(\theta)d_3d_7b^2-3sin(\theta)ad_{11}b^2-cos(\theta)d_3bd_{12}d_{16}\\
&-3sin(\theta)ab^2d_{15}-3sin(\theta)d_3ab^2-2cos(\theta)d_3abd_{15}\\
&+sin(\theta)d_3d_{11}b^2+sin(\theta)d_3b^2d_{15}+sin(\theta)d_7d_{11}b^2\\
&-sin(\theta)d_3ad_{12}d_{16}+sin(\theta)d_7b^2d_{15}+sin(\theta)b^2d_{11}d_{15}\\
&-sin(\theta)b^2d_{12}d_{16}-sin(\theta)d_8b^2d_{16}-sin(\theta)d_3d_7d_{11}d_{15}\\
&-sin(\theta)d_8d_{12}b^2-3sin(\theta)d_4a^2b-sin(\theta)d_4b^2d_{16}\\
&+2sin(\theta)d_3ad_{12}b-sin(\theta)d_4d_{12}b^2-sin(\theta)d_4d_8b^2+3d_1a^2d_9b\\
&+3d_1a^2bd_{13}+3d_1d_5a^2b-3d_1ab^2d_{14}-sin(\theta)d_4d_7d_{12}a\\
&+d_1d_5b^2d_{14}-3d_1ad_{10}b^2-3d_1ad_6b^2-2d_1abd_9d_{13}\\
&+d_1d_5d_{10}b^2+d_1b^2d_{14}d_9+d_1b^2d_{10}d_{13}+d_1d_6d_9b^2\\
&+d_1d_6b^2d_{13}-3d_2a^2bd_{14}-sin(\theta)d_4d_7d_{16}a\\
&-cos(\theta)d_3d_8d_{12}b+2sin(\theta)ad_8bd_{15}-3d_2a^2d_{10}b-3d_2d_6a^2b\\
&-sin(\theta)d_4d_8d_{11}a+cos(\theta)d_3d_8ad_{15}-d_2bd_5d_{14}d_9\\
&-d_2bd_5d_{10}d_{13}+cos(\theta)d_3bd_{11}d_{15}=0
\end{aligned}\tag{B12}$$

$$\begin{aligned}
&d_3d_7d_{16}d_{11}-d_4d_8d_{16}d_{11}-d_3d_8d_{12}d_{16}+d_3d_8d_{11}d_{15}\\
&-d_4d_7d_{12}d_{16}+d_4d_7d_{11}d_{15}+d_3d_7d_{12}d_{15}-d_3d_{12}b^2-d_8b^2d_{15}\\
&-b^2d_{16}d_{11}-d_3b^2d_{16}\ \ -d_7d_{12}b^2-d_3d_8b^2-d_7b^2d_{16}-b^2d_{12}d_{15}\\
&-d_4d_7b^2-d_4d_{11}b^2+d_1d_6b^3-d_8d_{11}b^2+d_1b^3d_{14}-d_4b^2d_{15}\\
&+d_1d_{10}b^3+d_2d_5b^3-d_3d_7d_{11}b-d_4d_8d_{12}d_{15}+d_2d_9b^3\\
&+d_2b^3d_{13}+d_3d_8bd_{16}+d_3d_8d_{12}b-bd_7d_{11}d_{15}+bd_7d_{12}d_{16}\\
&+bd_8d_{16}d_{11}+bd_8d_{12}d_{15}+d_4d_7bd_{16}+d_4d_7d_{12}b+d_4bd_{16}d_{11}\\
&-d_3d_7bd_{15}-d_3bd_{11}d_{15}+d_3bd_{12}d_{16}+d_4bd_{12}d_{15}+d_4d_8d_{11}b\\
&+d_4d_8bd_{15}+d_1d_5d_9b^2+d_1d_5b^2d_{13}+d_1b^2d_9d_{13}-d_1b^2d_{10}d_{14}\\
&-d_1d_6b^2d_{14}-d_1d_6d_{10}b^2-d_1bd_6d_9d_{13}-d_2d_5b^2d_{14}\\
&-d_2d_5d_{10}b^2-d_2b^2d_{14}d_9-d_2b^2d_{10}d_{13}-d_2d_6d_9b^2-d_2d_6b^2d_{13}\\
&-d_1bd_5d_{14}d_9-d_1bd_5d_{10}d_{13}+d_3b^3+d_7b^3+d_{11}b^3+b^3d_{15}\\
&-d_1b^4+d_1bd_6d_{10}d_{14}-d_2bd_5d_9d_{13}+d_2bd_5d_{10}d_{14}\\
&+d_2bd_6d_{14}d_9+d_2bd_6d_{10}d_{13}=0
\end{aligned}\tag{B13}$$

$$\begin{aligned}
&-d_3d_7bd_{16}-d_4d_8d_{12}d_{16}-d_3d_7d_{11}d_{15}-d_2b^4-d_3d_7d_{12}b\\
&+d_4d_7d_{16}d_{11}+d_4d_7d_{12}d_{15}+d_4d_8d_{11}d_{15}+d_2b^3d_{14}+d_3b^2d_{15}\\
&+d_2d_{10}b^3+d_3\qquad d_7b^2+d_7b^2d_{15}+d_3d_{11}b^2+d_7d_{11}b^2-d_1d_9b^3\\
&-d_4d_{12}b^2+b^2d_{11}d_{15}-b^2d_{12}d_{16}-d_8b^2d_{16}-d_4d_8b^2-d_4b^2d_{16}\\
&+d_2d_6b^3-d_1d_5b^3-d_3bd_{16}d_{11}-d_3bd_{12}d_{15}-d_3d_8d_{11}b\\
&-d_3d_8bd_{15}-bd_7d_{16}d_{11}-bd_7d_{12}d_{15}-bd_8d_{11}d_{15}\\
&+bd_8d_{12}d_{16}-d_4d_7d_{11}b-d_4d_7bd_{15}-d_4bd_{11}d_{15}+d_4bd_{12}d_{16}\\
&+d_4d_8bd_{16}+d_4d_8d_{12}b+b^3d_{16}+d_{12}b^3+d_8b^3+d_4b^3\\
&-d_1b^3d_{13}+d_2bd_6d_{10}d_{14}+d_3d_7d_{12}d_{16}-b^4-d_2bd_5d_{14}d_9\\
&-d_2bd_5d_{10}d_{13}-d_2bd_6d_9d_{13}+d_3d_8d_{16}d_{11}+d_3d_8d_{12}d_{15}\\
&+d_1bd_5d_9d_{13}-d_1bd_5d_{10}d_{14}-d_1bd_6d_{14}d_9-d_1bd_6d_{10}d_{13}\\
&+d_1d_5b^2d_{14}+d_1d_5d_{10}b^2-d_8d_{12}b^2+d_1b^2d_{14}d_9+d_1b^2d_{10}d_{13}\\
&+d_1d_6d_9b^2+d_1d_6b^2d_{13}+d_2d_5d_9b^2+d_2d_5b^2d_{13}+d_2b^2d_9d_{13}\\
&-d_2b^2d_{10}d_{14}-d_2d_6b^2d_{14}-d_2d_6d_{10}b^2=0
\end{aligned}\tag{B14}$$

$$cos(\theta)d_3a^3 - d_1d_9a^3 - d_1d_5a^3 + d_2d_{10}a^3 - d_1a^3d_{13} + d_2d_{14}a^3$$
$$- cos(\theta)d_4d_{12}a^2 + d_2a^2d_{10}d_{13} - cos(\theta)d_4b^3 - 6cos(\theta)a^2b^2$$
$$+ cos(\theta)d_{11}a^3 + sin(\theta)d_{12}a^3 + cos(\theta)d_7a^3 - cos(\theta)b^3d_{16}$$
$$+ d_1a^2d_{10}d_{14} + ad_1d_6d_{14}d_9 + ad_1d_6d_{10}d_{13} - d_1d_5d_9a^2$$
$$+ sin(\theta)d_3ad_{12}d_{15} - cos(\theta)d_4d_8d_{11}a - cos(\theta)d_4d_8d_{11}d_{15}$$
$$+ cos(\theta)d_4d_8d_{12}d_{16} + 2cos(\theta)ad_7bd_{16} + d_2d_5d_{10}a^2 + d_2d_6a^2d_{13}$$
$$+ cos(\theta)a^2d_{11}d_{15} - cos(\theta)a^2d_{12}d_{16} + cos(\theta)a^3d_{15}$$
$$+ cos(\theta)d_7a^2d_{15} - cos(\theta)d_8d_{16}a^2 - cos(\theta)d_8d_{12}a^2$$
$$+ cos(\theta)d_3a^2d_{15} + cos(\theta)d_3d_{11}a^2 + cos(\theta)d_3d_7a^2$$
$$+ sin(\theta)bd_8d_{16}d_{11} + sin(\theta)bd_8d_{12}d_{15} + cos(\theta)bd_8d_{11}d_{15}$$
$$- cos(\theta)bd_8d_{12}d_{16} + 2cos(\theta)d_4ad_{11}b + 2cos(\theta)d_4abd_{15}$$
$$+ sin(\theta)d_3d_7d_{16}a + sin(\theta)d_3d_7d_{16}d_{11} + sin(\theta)d_3d_7d_{12}a$$
$$- 3sin(\theta)d_3a^2b - sin(\theta)d_3b^2d_{16} + sin(\theta)d_4d_7d_{11}a$$
$$+ sin(\theta)d_4d_7d_{11}d_{15} - sin(\theta)d_4d_7d_{12}d_{16} - cos(\theta)d_4d_8ad_{15}$$
$$+ 2sin(\theta)abd_{12}d_{16} - 2sin(\theta)ad_7bd_{15} - 2sin(\theta)abd_{11}d_{15}$$
$$- d_1bd_5d_{10}d_{13} - d_1bd_6d_9d_{13} + sin(\theta)d_3d_8ad_{15} + sin(\theta)d_3d_8d_{11}a$$
$$- 3cos(\theta)d_3ab^2 - cos(\theta)d_3d_7b^2 - cos(\theta)d_3d_{11}b^2$$
$$- cos(\theta)d_3b^2d_{15} - cos(\theta)d_7d_{11}b^2 + cos(\theta)d_3bd_{12}d_{15}$$
$$+ 2cos(\theta)d_3d_8ab + cos(\theta)d_3d_8d_{11}b - sin(\theta)d_7b^2d_{16}$$
$$- sin(\theta)d_8d_{11}b^2 + cos(\theta)d_7d_{11}a^2 - 2d_1abd_{14}d_9$$
$$- 2d_1abd_{10}d_{13} - ad_2d_6d_{10}d_{14} + ad_2d_6d_9d_{13}$$
$$+ 2sin(\theta)d_4d_8ab + sin(\theta)d_4d_8d_{11}b - d_1d_6b^2d_{14} - d_1d_6d_{10}b^2$$
$$- 3d_2a^2d_9b - 3cos(\theta)ab^2d_{15} - d_2d_5b^2d_{14} - d_2d_5d_{10}b^2$$
$$+ 3cos(\theta)d_4a^2b + cos(\theta)d_8b^2d_{16} + cos(\theta)d_8d_{12}b^2$$
$$- 3d_2ad_6b^2 - 3d_2ab^2d_{14} - 3d_2ad_{10}b^2 - sin(\theta)bd_7d_{11}d_{15}$$
$$- cos(\theta)d_{12}b^3 + d_2d_5d_{14}a^2 - d_1a^2d_9d_{13} + sin(\theta)d_4d_7ad_{15}$$
$$- ad_1d_5d_9d_{13} + ad_1d_5d_{10}d_{14} - d_2d_6b^2d_{13} - sin(\theta)d_4b^2d_{15}$$
$$- 3d_1a^2bd_{14} - cos(\theta)d_4d_{16}a^2 + d_2bd_5d_{10}d_{14} + sin(\theta)d_4a^3$$
$$+ 3d_1ad_9b^2 + 3d_1ab^2d_{13} + d_1d_5d_9b^2 + sin(\theta)d_3d_8d_{12}b$$
$$+ sin(\theta)d_3bd_{12}d_{16} + sin(\theta)d_3d_8bd_{16} + 2cos(\theta)d_3abd_{16}$$
$$+ 2cos(\theta)d_3ad_{12}b + cos(\theta)d_3d_7bd_{16} + cos(\theta)d_3d_7d_{12}b$$
$$+ 2sin(\theta)d_4abd_{16} + 2sin(\theta)d_4ad_{12}b - d_1b^2d_{10}d_{14} + d_1d_5b^2d_{13}$$
$$+ d_1b^2d_9d_{13} + cos(\theta)d_3d_8bd_{15} + cos(\theta)bd_7d_{16}d_{11}$$
$$+ cos(\theta)bd_7d_{12}d_{15} - sin(\theta)d_4d_8d_{12}a - sin(\theta)d_4d_8d_{16}a$$
$$- sin(\theta)d_4d_8d_{16}d_{11} - 2d_2abd_9d_{13} + 2d_2abd_{10}d_{14}$$
$$+ cos(\theta)ad_7d_{11}d_{15} - cos(\theta)ad_7d_{12}d_{16} - cos(\theta)ad_8d_{16}d_{11}$$
$$- d_2b^2d_{14}d_9 - d_2b^2d_{10}d_{13} - sin(\theta)ad_8d_{12}d_{16} + sin(\theta)d_3ad_{16}d_{11}$$
$$+ 2cos(\theta)ad_7d_{12}b + 2cos(\theta)abd_{16}d_{11} + 2cos(\theta)abd_{12}d_{15}$$
$$+ 2cos(\theta)ad_8d_{11}b + sin(\theta)d_{16}a^3 - cos(\theta)d_8b^3 - sin(\theta)d_4d_{11}b^2$$
$$+ cos(\theta)b^4 - d_1b^4 - d_1d_5a^2d_{13} + 3cos(\theta)a^2bd_{16}$$
$$+ 3cos(\theta)a^2d_{12}b - cos(\theta)ad_8d_{12}d_{15} + cos(\theta)d_3ad_{11}d_{15}$$
$$- cos(\theta)d_3ad_{12}d_{16} + cos(\theta)d_3d_7ad_{15} + cos(\theta)d_3d_7d_{11}a$$
$$+ cos(\theta)d_3d_7d_{11}d_{15} + 2d_2ad_6d_{10}b + d_1bd_6d_{10}d_{14}$$
$$- sin(\theta)d_7d_{12}b^2 - sin(\theta)b^2d_{16}d_{11} - sin(\theta)b^2d_{12}d_{15}$$
$$- 3sin(\theta)ad_{12}b^2 - 3sin(\theta)ad_8b^2 + sin(\theta)d_3b^3$$
$$-d_2bd_5d_9d_{13} - cos(\theta)d_4d_8a^2 - 2d_2ad_5bd_{13} + d_2a^2d_{14}d_9 - d_1a^4$$
$$+ d_1d_6b^3 + sin(\theta)d_8a^3 - 2d_1ad_6d_9b + d_2d_5b^3 - 3d_1a^2d_{10}b$$
$$- 3d_1d_6a^2b + 3d_1ad_5b^2 - 4sin(\theta)a^3b + sin(\theta)d_8a^2d_{15}$$
$$+ sin(\theta)a^2d_{12}d_{15} + sin(\theta)d_7d_{16}a^2 + sin(\theta)d_4a^2d_{15}$$
$$+ sin(\theta)d_4d_{11}a^2 - sin(\theta)d_3d_7bd_{15} - sin(\theta)d_3bd_{11}d_{15}$$
$$+ d_2d_6a^3 - 3d_2a^2bd_{13} - 3d_2d_5a^2b + d_1d_6d_{10}a^2$$
$$-cos(\theta)d_7b^2d_{15} - cos(\theta)b^2d_{11}d_{15} + cos(\theta)b^2d_{12}d_{16} + d_1d_6d_{14}a^2$$
$$+ 4sin(\theta)ab^3 - 3cos(\theta)ad_{11}b^2 + 3cos(\theta)d_8a^2b - 3cos(\theta)ad_7b^2$$
$$+ sin(\theta)d_{11}b^3 + sin(\theta)d_7b^3 + d_2d_6d_9a^2 - sin(\theta)d_4d_8d_{12}d_{15}$$
$$- 2sin(\theta)ad_7d_{11}b + d_1b^3d_{14} + 2d_2ad_6bd_{14} - 4d_2a^3b + d_1d_{10}b^3$$
$$+ 6d_1a^2b^2 + sin(\theta)b^3d_{15} - sin(\theta)d_4ad_{12}d_{16} + sin(\theta)d_4d_8bd_{15}$$
$$- 2d_1ad_5bd_{14} - sin(\theta)d_3d_8d_{12}d_{16} + sin(\theta)d_4ad_{11}d_{15}$$
$$- d_1bd_5d_{14}d_9 - cos(\theta)d_4d_8d_{12}b - cos(\theta)d_4d_8bd_{16} - d_2d_6d_9b^2$$
$$+ sin(\theta)d_3d_8d_{11}d_{15} + sin(\theta)d_4d_7bd_{16} + sin(\theta)d_4d_7d_{12}b$$
$$- 2sin(\theta)d_3d_7ab - sin(\theta)d_3d_7d_{11}b + ad_2d_5d_{14}d_9 + ad_2d_5d_{10}d_{13}$$
$$+ 2cos(\theta)d_4d_7ab + cos(\theta)d_4d_7d_{11}b + cos(\theta)d_4d_7bd_{15}$$
$$- 2sin(\theta)d_3ad_{11}b - 2sin(\theta)d_3abd_{15} - 2d_1ad_6bd_{13} + d_2b^3d_{13}$$

$$
\begin{aligned}
&+4d_2ab^3+d_2d_9b^3+cos(\theta)a^4+cos(\theta)d_3bd_{16}d_{11}\\
&+sin(\theta)d_3d_7d_{12}d_{15}+2cos(\theta)ad_8bd_{15}+sin(\theta)bd_7d_{12}d_{16}\\
&+d_2bd_6d_{10}d_{13}+d_2bd_6d_{14}d_9-sin(\theta)d_8b^2d_{15}-3sin(\theta)d_4ab^2\\
&-sin(\theta)d_4d_7b^2+cos(\theta)d_4d_8b^2+sin(\theta)d_4d_7a^2-sin(\theta)d_3d_{12}b^2\\
&-sin(\theta)d_3d_8b^2+sin(\theta)d_7d_{12}a^2+sin(\theta)a^2d_{16}d_{11}\\
&+sin(\theta)d_8d_{11}a^2+sin(\theta)d_3d_8a^2-cos(\theta)d_3d_7d_{12}d_{16}\\
&-cos(\theta)d_3d_8d_{16}a-cos(\theta)d_3d_8d_{16}d_{11}-cos(\theta)d_3d_8d_{12}a\\
&-cos(\theta)d_3d_8d_{12}d_{15}-cos(\theta)d_4ad_{16}d_{11}+sin(\theta)ad_7d_{16}d_{11}\\
&+cos(\theta)d_4b^2d_{16}+cos(\theta)d_4d_{12}b^2+sin(\theta)d_3d_{12}a^2\\
&+sin(\theta)d_3d_{16}a^2-cos(\theta)d_4ad_{12}d_{15}-cos(\theta)d_4d_7d_{16}a\\
&-cos(\theta)d_4d_7d_{16}d_{11}-cos(\theta)d_4d_7d_{12}a-cos(\theta)d_4d_7d_{12}d_{15}\\
&+2sin(\theta)ad_8bd_{16}+2sin(\theta)ad_8d_{12}b+sin(\theta)d_4bd_{16}d_{11}\\
&+sin(\theta)d_4bd_{12}d_{15}-3sin(\theta)a^2d_{11}b-3sin(\theta)a^2bd_{15}\\
&-3sin(\theta)d_7a^2b-3sin(\theta)ab^2d_{16}+sin(\theta)ad_7d_{12}d_{15}\\
&+sin(\theta)ad_8d_{11}d_{15}-2d_2ad_5d_9b-2d_1ad_5d_{10}b\\
&+cos(\theta)d_4bd_{11}d_{15}-cos(\theta)d_4bd_{12}d_{16}=0 \qquad \text{(B15)}
\end{aligned}
$$

$$
\begin{aligned}
&-d_1a^2d_{14}d_9-ad_1d_5d_{14}d_9-4cos(\theta)a^3b+sin(\theta)d_{12}b^3-4d_1ab^3\\
&+6d_2a^2b^2-d_2d_9a^3-d_1d_{10}a^3-d_2a^3d_{13}-d_1d_6a^3\\
&-ad_1d_6d_9d_{13}-d_2d_5a^3-d_1d_{14}a^3-ad_1d_5d_{10}d_{13}+sin(\theta)b^3d_{16}\\
&+cos(\theta)d_{12}a^3-d_1d_6a^2d_{13}-d_1d_5d_{10}a^2-d_1d_5d_{14}a^2\\
&-d_1a^2d_{10}d_{13}+cos(\theta)d_3b^3-d_1d_9b^3+sin(\theta)d_4b^3\\
&+ad_2d_5d_{10}d_{14}-ad_2d_5d_9d_{13}-d_1d_5b^3+4cos(\theta)ab^3\\
&-sin(\theta)d_{11}a^3+cos(\theta)d_4a^3+d_2d_{10}b^3+cos(\theta)d_7b^3\\
&-d_2d_5a^2d_{13}+3d_2ad_5b^2+3d_2ad_9b^2+d_2a^2d_{10}d_{14}\\
&-d_2a^2d_9d_{13}-d_1d_6d_9a^2+3d_2ab^2d_{13}+ad_1d_6d_{10}d_{14}\\
&+cos(\theta)d_{16}a^3+cos(\theta)d_8a^3+d_2d_6b^3-sin(\theta)d_3a^3\\
&+sin(\theta)d_8b^3-sin(\theta)a^3d_{15}+ad_2d_6d_{10}d_{13}+ad_2d_6d_{14}d_9\\
&+3d_1a^2d_9b+3d_1a^2bd_{13}+3d_1d_5a^2b+3d_1ab^2d_{14}+3d_1ad_{10}b^2\\
&+3d_1ad_6b^2-3d_2a^2bd_{14}-3d_2a^2d_{10}b-3d_2d_6a^2b\\
&+3sin(\theta)ab^2d_{15}+3sin(\theta)d_3ab^2-3sin(\theta)d_4a^2b-d_2d_5d_9a^2\\
&-d_2a^4-3sin(\theta)a^2d_{12}b-3sin(\theta)d_8a^2b+3sin(\theta)ad_7b^2\\
&+3sin(\theta)ad_{11}b^2+6sin(\theta)a^2b^2+d_1bd_5d_9d_{13}+d_2d_6d_{10}a^2\\
&+d_2d_6d_{14}a^2-3sin(\theta)a^2bd_{16}-d_1bd_6d_{10}d_{13}-d_2bd_6d_9d_{13}\\
&-2d_1ad_6d_{10}b+cos(\theta)ad_7d_{16}d_{11}-2d_1ad_6bd_{14}\\
&+cos(\theta)d_3ad_{12}d_{15}+cos(\theta)d_3ad_{16}d_{11}-cos(\theta)ad_8d_{12}d_{16}\\
&+cos(\theta)ad_8d_{11}d_{15}+cos(\theta)ad_7d_{12}d_{15}-cos(\theta)d_3d_8d_{12}d_{16}\\
&+cos(\theta)d_3d_8d_{11}d_{15}+cos(\theta)d_3d_8d_{11}a+cos(\theta)d_3d_8ad_{15}\\
&+cos(\theta)d_3d_7d_{12}d_{15}+cos(\theta)d_3d_7d_{12}a+cos(\theta)d_3d_7d_{16}d_{11}\\
&+cos(\theta)d_3d_7d_{16}a-3cos(\theta)a^2d_{11}b-3cos(\theta)a^2bd_{15}\\
&-3cos(\theta)d_7a^2b-3cos(\theta)ab^2d_{16}-3cos(\theta)ad_{12}b^2\\
&-3cos(\theta)ad_8b^2-3cos(\theta)d_3a^2b-3cos(\theta)d_4ab^2-2d_2ad_5d_{10}b\\
&-d_2d_6b^2d_{14}-d_2d_6d_{10}b^2-2d_1abd_{10}d_{14}+sin(\theta)d_7b^2d_{15}\\
&+sin(\theta)b^2d_{11}d_{15}-sin(\theta)b^2d_{12}d_{16}-sin(\theta)d_8b^2d_{16}\\
&-sin(\theta)d_8d_{12}b^2-sin(\theta)d_4b^2d_{16}-sin(\theta)d_4d_{12}b^2-sin(\theta)d_4d_8b^2\\
&+d_1d_5b^2d_{14}+d_1d_5d_{10}b^2+d_1b^2d_{14}d_9+d_1b^2d_{10}d_{13}\\
&+d_1d_6d_9b^2+d_1d_6b^2d_{13}+d_2d_5d_9b^2+d_2d_5b^2d_{13}\\
&+d_2b^2d_9d_{13}-d_2b^2d_{10}d_{14}-d_1bd_5d_{10}d_{14}+cos(\theta)d_4ad_{11}d_{15}\\
&+cos(\theta)d_4d_{11}a^2-cos(\theta)d_3b^2d_{16}+cos(\theta)d_4a^2d_{15}\\
&-cos(\theta)d_3d_{12}b^2-cos(\theta)d_3d_8b^2-cos(\theta)d_7b^2d_{16}\\
&-cos(\theta)d_7d_{12}b^2-cos(\theta)b^2d_{16}d_{11}-cos(\theta)b^2d_{12}d_{15}\\
&-cos(\theta)d_8d_{11}b^2-cos(\theta)d_8b^2d_{15}-cos(\theta)d_4d_7b^2\\
&-cos(\theta)d_4d_{11}b^2-cos(\theta)d_4b^2d_{15}-sin(\theta)a^2d_{11}d_{15}\\
&+sin(\theta)a^2d_{12}d_{16}-sin(\theta)d_7a^2d_{15}-sin(\theta)d_7d_{11}a^2\\
&+sin(\theta)d_8d_{16}a^2+sin(\theta)d_8d_{12}a^2-sin(\theta)d_3a^2d_{15}\\
&-sin(\theta)d_3d_{11}a^2-sin(\theta)d_3d_7a^2+sin(\theta)d_4d_{16}a^2\\
&+sin(\theta)d_4d_{12}a^2+sin(\theta)d_4d_8a^2+sin(\theta)d_3d_7b^2+sin(\theta)d_3d_{11}b^2\\
&+sin(\theta)d_3b^2d_{15}+sin(\theta)d_7d_{11}b^2-sin(\theta)d_3bd_{12}d_{15}\\
&-sin(\theta)d_3bd_{16}d_{11}-sin(\theta)d_3d_7d_{12}b-sin(\theta)d_3d_7bd_{16}\\
&-sin(\theta)d_4d_8d_{12}d_{16}+sin(\theta)d_4d_8d_{11}d_{15}+sin(\theta)d_4d_8d_{11}a\\
&+sin(\theta)d_4d_8ad_{15}+sin(\theta)d_4d_7d_{12}d_{15}+sin(\theta)d_4d_7d_{12}a\\
&+sin(\theta)d_4d_7d_{16}d_{11}+sin(\theta)d_4d_7d_{16}a+sin(\theta)d_4ad_{12}d_{15}
\end{aligned}
$$

Industrial Lubrication and Tribology
Volume 70 · Number 8 · 2018 · 1457–1475

$$
\begin{aligned}
&+ sin(\theta)d_4ad_{16}d_{11} + sin(\theta)d_3d_8d_{12}d_{15} + sin(\theta)d_3d_8d_{12}a \\
&+ sin(\theta)d_3d_8d_{16}d_{11} + sin(\theta)d_3d_8d_{16}a + sin(\theta)d_3d_7d_{12}d_{16} \\
&- sin(\theta)d_3d_7d_{11}d_{15} - sin(\theta)d_3d_7d_{11}a - sin(\theta)d_3d_7ad_{15} \\
&+ sin(\theta)d_3ad_{12}d_{16} - sin(\theta)d_3ad_{11}d_{15} + sin(\theta)ad_8d_{12}d_{15} \\
&+ sin(\theta)ad_8d_{16}d_{11} + sin(\theta)ad_7d_{12}d_{16} - sin(\theta)ad_7d_{11}d_{15} \\
&+ cos(\theta)d_4d_8bd_{15} + cos(\theta)d_4d_8d_{11}b + cos(\theta)d_4bd_{12}d_{15} \\
&+ cos(\theta)d_4bd_{16}d_{11} + cos(\theta)d_4d_7d_{12}b + cos(\theta)d_4d_7bd_{16} \\
&+ cos(\theta)bd_8d_{12}d_{15} + cos(\theta)bd_8d_{16}d_{11} + cos(\theta)bd_7d_{12}d_{16} \\
&- cos(\theta)bd_7d_{11}d_{15} + cos(\theta)d_3d_8d_{12}b + cos(\theta)d_3d_8bd_{16} \\
&+ cos(\theta)d_3bd_{12}d_{16} - cos(\theta)d_3bd_{11}d_{15} - cos(\theta)d_3d_7bd_{15} \\
&- cos(\theta)d_3d_7d_{11}b - cos(\theta)d_4d_8d_{12}d_{15} - cos(\theta)d_4d_8d_{12}a \\
&- cos(\theta)d_4d_8d_{16}d_{11} - cos(\theta)d_4d_8d_{16}a - cos(\theta)d_4d_7d_{12}d_{16} \\
&+ cos(\theta)d_4d_7d_{11}d_{15} + cos(\theta)d_4d_7d_{11}a + cos(\theta)d_4d_7ad_{15} \\
&- cos(\theta)d_4ad_{12}d_{16} + sin(\theta)bd_8d_{12}d_{16} - sin(\theta)d_3d_8bd_{15} \\
&- sin(\theta)d_4d_7d_{11}b + 2d_1ad_5bd_{13} + d_2bd_6d_{10}d_{14} \\
&- sin(\theta)d_4bd_{11}d_{15} + cos(\theta)d_3d_{16}a^2 + cos(\theta)b^3d_{15} - 2d_2ad_6d_9b \\
&- sin(\theta)d_3d_8d_{11}b - d_1b^3d_{13} + 4d_1a^3b - 2cos(\theta)abd_{11}d_{15} \\
&- 2cos(\theta)ad_7bd_{15} - 2cos(\theta)ad_7d_{11}b + 2cos(\theta)abd_{12}d_{16} \\
&- 2cos(\theta)d_3d_7ab - 2cos(\theta)d_3abd_{15} - 2cos(\theta)d_3ad_{11}b \\
&+ 2cos(\theta)ad_8d_{12}b + 2cos(\theta)ad_8bd_{16} + 2cos(\theta)d_4abd_{16} \\
&- 2sin(\theta)d_4d_7ab + 2cos(\theta)d_4ad_{12}b - 2d_2abd_{14}d_9 \\
&+ 2cos(\theta)d_4d_8ab - sin(\theta)d_7a^3 - 2sin(\theta)abd_{16}d_{11} \\
&- sin(\theta)bd_8d_{11}d_{15} - 2sin(\theta)d_4abd_{15} - 2sin(\theta)ad_7d_{12}b \\
&- 2sin(\theta)ad_7bd_{16} - 2sin(\theta)d_3abd_{16} - 2sin(\theta)ad_8bd_{15} \\
&- 2sin(\theta)ad_8d_{11}b - 2sin(\theta)abd_{12}d_{15} + cos(\theta)d_8d_{11}a^2 \\
&+ cos(\theta)a^2d_{16}d_{11} + cos(\theta)d_7d_{12}a^2 - 2sin(\theta)d_3ad_{12}b \\
&+ cos(\theta)d_3d_8a^2 - 2sin(\theta)d_3d_8ab - d_2bd_5d_{10}d_{13} \\
&- 2sin(\theta)d_4ad_{11}b - sin(\theta)bd_7d_{12}d_{15} - d_2bd_5d_{14}d_9 \\
&- 2d_2ad_5bd_{14} + cos(\theta)d_{11}b^3 + d_2b^3d_{14} + sin(\theta)d_4bd_{12}d_{16} \\
&- d_2b^4 + 2d_1abd_9d_{13} + sin(\theta)d_4d_8bd_{16} + 2d_1ad_5d_9b \\
&- 2d_2abd_{10}d_{13} - 2d_2ad_6bd_{13} + cos(\theta)d_3d_{12}a^2 \\
&- sin(\theta)bd_7d_{16}d_{11} + sin(\theta)d_4d_8d_{12}b - d_1bd_6d_{14}d_9 - sin(\theta)a^4 \\
&- sin(\theta)b^4 + cos(\theta)d_7d_{16}a^2 + cos(\theta)a^2d_{12}d_{15} + cos(\theta)d_8a^2d_{15} \\
&+ cos(\theta)d_4d_7a^2 - sin(\theta)d_4d_7bd_{15} = 0 \qquad \text{(B16)}
\end{aligned}
$$

Corresponding author

Shang-Han Gao can be contacted at: gxgshh@163.com