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The expansion and stability of two-sided platform ecological networks: an analysis based on the leverage theory of tying

Zhang Qian
School of Economics and Trade, Hunan University of Technology and Business, Changsha, China
Cui Wei
National University of Singapore, Singapore, Singapore, and
Tang Chao and Luo Yan
School of Economics and Trade, Hunan University, Changsha, China

Abstract

Purpose – With the rapid development of the digital economy, an increasing number of digitalized two-sided platforms have deployed the tying strategy to leverage their market power from the core two-sided product to other two-sided products in the competitive market, which transforms the competition among single platforms into that among platform ecological networks. To clarify the mechanism of the formation of the digital platform ecological networks, this paper aims to analyze the expansion and stability of platform ecology by exploring the impacts of network externalities and sellers' heterogeneity on the tying strategy of two-sided platforms.

Design/methodology/approach – This paper develops a game model of two-sided platforms based on Choi and Jeon (2021), which highlights the decisive influence of non-negative price constraints (NPC) on platforms' tying motivation. Taking the operating systems market as an example, we expand from the perspective of platform service differences to relax the NPC and explore the internal logic of platform ecosystem expansion.

Findings – Platforms have an incentive to charge lower prices or even subsidize buyers when the network externalities on the sellers' side are relatively strong. When the product is highly differentiated and heterogenous, platforms are motivated to tie to capture more buyers with a lower price and grab excess profits from sellers. Eventually, tying is able to consolidate the two-sided platform ecological networks by excluding competitors, capturing user value and deterring entry.

Originality/value – In order to describe the characteristics of platform ecological network more generally, this paper extends the research based on the analyses of Choi and Jeon (2021) by (1) allowing horizontal differences between tied products and (2) relaxing the NPC. Unlike Choi and Jeon (2021), this paper allows platforms to charge users of two-sided platforms at negative prices (or to subsidize them). (3) Setting simultaneous pricing in two-sided platforms. Classical two-sided market theory stresses that the presence of cross-network externalities can give rise to a "chicken and egg" problem.

Keywords Two-sided platforms, Tying, Leverage theory, Business ecosystem, Dynamic game model

Paper type Research paper

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1. Introduction

With the rapid development of the digital economy and the accelerating trend toward the platform of firms, an increasing number of digitalized two-sided platforms have deployed the tying strategy to leverage their market power from a core two-sided product to other two-sided products in the competitive market. This transforms the competition among single platforms into that among platform ecological networks, which aims to meet the diversified and one-stop needs of users. This trend is extremely prominent in the operating system market. For example, Microsoft has bundled products such as Internet Explorer, Windows Media Player and Microsoft Store with its operating system to expand its business landscape. Similarly, Google requires cell phone manufacturers to pre-install search engines, Google Chrome, Google Maps and other applications in the Android operating system as a prerequisite for using its Google Mobile Services and Google Play.

To explore the internal mechanism of the formation and development of the platform ecological network, scholars have conducted extensive research from the perspectives of product complementarity, network externalities and user inertia. Considering the complementarity or interoperability of digital products, some researchers argue that the key to the continuous expansion of platforms and the formation of business ecological networks lies in the fact that different products in the ecological network are complementary, enabling buyers and sellers to realize value co-creation through the interaction of multiple products (Ceccagnoli, Forman, Huang, & Wu, 2012). Others emphasize that (cross) network externalities as key features that distinguish digital platforms from traditional firms can not only help accumulate loyal users and increase user value, but also facilitate digital platforms to build core competencies and gain a competitive edge in a "*winner-take-all*" market (Armstrong, 2006; Song, Xue, Rai, & Zhang, 2018). Still, others, looking at the problem from the perspectives of buyer inertia and transfer costs, point out that the evolution of platform ecological networks stems from the firms with the "*first-mover advantage*," which enhance buyers' brand loyalty by locking-in consumer behavior or consumer inertia (Shi, Lin, Liu, & Hui, 2018). Indeed, these studies are helpful for us to understand the emergence of platform ecological networks from multiple perspectives. However, the issue of the rationality of the platforms' behavioral strategy in building the ecological network is insufficiently discussed since their analyses largely focus on each platform itself. To our knowledge, Eisenmann, Parker, and Van Alstyne (2011) are the first to have used the "*Platform Envelopment Theory*" to summarize the behaviors of platform-based firms that use the tying strategy to build their own ecosystems, which categorizes the ecological networks of platforms based on the correlation between different two-sided platforms with the case study. In order to clarify and generalize the mechanism of the formation of the digital platform ecological networks, we use economic theories of two-sided markets, starting from the incentive for the platforms to tie, to do an in-depth analysis of the formation and expansion of platform ecological networks by putting the product and platform characteristics and user behavior patterns into a unified analytical framework, which is closely related to the theoretical research of Choi and Jeon (2021) and Iacobucci and Ducci (2019). Meanwhile, we explore the stability of the business ecosystem and its evolutionary logic from the aspects of market competition, potential entry, etc.

Relevant research mainly involves platform ecology, network externality and tying, as is shown in Table 1. The digital economy features strong network externalities, extremely low marginal cost and high capital turnover, which have made the two-sided platforms the main direction of business model innovation for digital firms. Two-sided platforms refer to intermediaries or platforms that provide transaction services to different types of users, and the utility level of one type of users is affected by the number of other users, i.e. there exists (cross) network externalities (Armstrong, 2006; Rochet & Tirole, 2003). Thus, platforms shift value from products and services to interactions, and ecosystems promote value through these interactions (Nichol, 2016). From this perspective, two-sided platform ecological network is a

Study	Category	Assumptions	The objective
Ceccagnoli *et al.* (2012)	Platform ecological network	Salient unintended knowledge spillovers	Value co-creation and appropriation are not mutually exclusive strategies in interfirm collaboration
Mariotto and Verdier (2015)		Tying the add-on service to their primary product	Platforms' incentive to foreclosure firms' tying
Chao and Derdenger (2013)	Platform with network externalities	Install the base with differentiated service	Bundling acts as a price discrimination tool segmenting the market more efficiently, and the two sides are better coordinated, and social welfare is enhanced
Iacobucci and Ducci (2019)	Tying by platform with network externalities	Prohibition of rebates	Tying of two-sided platforms may lead to the failure of the single monopoly profit theorem
Choi and Jeon (2021)		NPC; homogeneous products	Tying is profitable for two-sided platforms when there exist ancillary revenues that cannot be competed away in the tied good market
This paper		Subsidy or negative price is allowed; different products within the platform retain independent reserve value and different services are provided between platforms	Value co-creation and appropriation is clarified from network externality and tying, and the internal logic of platform ecological expansion is revealed

Table 1.
Context of literature review

platform network that integrates and coordinates different two-sided platforms to satisfy a series of online activities, which can deeply integrate products or services that previously belonged to different platforms into one product, such as user retrieval, shopping and social networking. Driven by deep specialization, subcontracting and the Internet of Things (*IoT*), the survival mode of platforms gradually changes from the competition of products and services to the competition among ecosystems (Tiwana, 2013). The platform ecosystem generates higher demands on the operation of digital firms, requiring them to balance platform functions and strategies. As pointed out by Benitez, Arenas, Castillo, and Esteves (2022), digital leadership improves a firm's innovation performance by digitalizing the firm's platform.

Tying refers to the practice of selling a product (the primary product) and asking customers to buy another product (the tied product) at the same time [1]. The typical difference between tying and other market expansion strategies is that the former is more flexible and is no longer limited to complementary products, i.e. tying can still play a role in products that are independent or substitutable (Armstrong, 2013; Belleflamme, 2005) [2]. To measure the impact of tying, the products that are tied are generally defined as value-independent products. Previous research on the tying theory has focused on the potential anti-competitive effects of tying in terms of price discrimination (Chao & Derdenger, 2013; Adams & Yellen, 1976), leverage theory (Whinston, 1990), entry deterrence (Nalebuff, 2004) and product differentiation (Chen, 1997). Table A1 in Appendix 1 summarizes related studies about firms' motivation of tying. The leverage theory of tying is most relevant to the construction of the ecological network when considering the impact of network externalities. The leverage theory of tying emphasizes that by integrating different products offered by two-sided platforms, firms can, therefore, gain a competitive advantage in the market of additional products by tying and expand that advantage to build a business ecological network through continuously integrating various products and services. As highlighted by Holzweber (2018), tying is particularly applicable to two-sided platforms. Rochet and Tirole (2003) take the Honor-All-Cards rules for bank cards as an example to stress that tying provides a strategy to balance the two-sided pricing in two-sided platforms, thus raising both platforms' benefits and social welfare. Choi (2010) explores the incentives to tying on two-sided platforms and its effects on social welfare from the perspective of buyers' multi-homing and argues that tying can boost market demand by increasing the proportion of multi-homing buyers, thus enhancing platforms' profit and social welfare. Mariotto and Verdier (2015) examine the incentive of two-sided platforms to use exclusive contracts when firms are able to tie additional services and point out that the said incentive depends mainly on buyers' acceptance of the primary product. In that connection, tying can help increase buyers' acceptance and merchants' engagement, such that the two-sided platforms need to trade-off between increasing consumer demand and merchant engagement. Iacobucci and Ducci (2019) examine the relationship between network externalities and tying strategies of two-sided platforms by conducting a case study of the Google search case in Europe. Choi and Jeon (2021) analyze the leverage theory of tying through the charging model of two-sided platforms and argue that the presence of the non-negative price constraint (NPC) allows two-sided platforms to be able to leverage their market power in the primary product market to the tied product market through tying. Regarding the expansion model of platform networks, relevant studies focus on the basic users (Chao & Derdenger, 2013; Jocevski, Ghezzi, & Arvidsson, 2020; Zhang, 2020), the mechanisms of consumer transformation between transactional and social behaviors (Li & Ku, 2018), bundling strategies of content providers (Raghunathan & Sarkar, 2016), two-sided user expectation changes (Hagiu & Spulber, 2013), etc. In addition, Foerderer (2020) uses the Apple Worldwide Developers Conference (WWDC 2016) as an impact of event and employs a Difference-in-Difference (DID) design to examine the mechanisms by which inter-firm exchanges influence complementary innovation and the firms' success and emphasizes that the opportunity to exchange will stimulate innovation in ecosystems.

In summary, the incentives for two-sided platforms to tie and the consequent impact on market competition and social welfare will become more uncertain due to network externalities. In particular, the tying strategy of two-sided platforms may still be effective without considering the influence of external efficiency factors such as economies of scale and R&D and thus significantly different from the tying strategy of traditional firms. At the same time, the research is still insufficient in terms of the tying strategy of two-sided platforms in strengthening their market competitiveness and consolidating the two-sided platform ecological networks. Theoretical studies, in particular, are scarce, regarding the analysis of the leverage mechanism of tying from product design and individual behavior.

In order to describe the characteristics of platform ecological networks more generally, this paper extends the research based on the analyses of Choi and Jeon (2021) and Iacobucci and Ducci (2019) by (1) allowing horizontal differences between tied products. Choi and Jeon (2021) assume that the products in the tied product market are homogeneous and that the quality of tied products on monopoly platforms is lower, i.e. we assume that vertical differences between products exist. Data show that Google Search demonstrates a 95% market share when it is set as the default search engine for Android devices, but that figure sinks to less than 25% when it is not the default (e.g. in a Windows device). This shows the rather large user base that Google Search has despite the fact that default installation is a key factor influencing consumer choice, in that its market share without pre-installation (25%) is much higher than that of all other search services combined when pre-installed (5%) (de Cornière & Taylor, 2018). Therefore, considering the differences in functions and appearance of different products, this paper focuses more on the horizontal differences between products. (2) Relaxing the non-negative price constraint. Unlike Choi and Jeon (2021), this paper allows platforms to charge users of two-sided platforms at negative prices (or to subsidize them). In practice, two-sided platforms often give users various additional services or cash incentives, such as cash bonuses, coupons, etc., in order to induce them to join. Also, this assumption helps to combine the studies of Choi and Jeon (2021) and Iacobucci and Ducci (2019) to examine the mechanisms in detail by which the leverage theory of tying works in two-sided markets. (3) Setting simultaneous pricing in two-sided platforms. Classical two-sided market theory stresses that the presence of cross-network externalities can give rise to a "chicken and egg" problem, where a user is motivated to participate in a platform only if she expects enough users from the other side to participate (Caillaud & Jullien, 2003). In contrast, in Choi and Jeon's (2021) analysis, the timing of users from the two sides is inconsistent. Therefore, despite the computational difficulty, we assume that the pricing decisions of the users on two-sided markets are made simultaneously.

This paper shows that: (1) the pricing by two-sided platforms for users on both sides is mainly determined by the relative strength of network externalities on the two sides of Market B, whereas the incentive to tie is mainly influenced by the intensity of the aggregate network externalities of both sides in Market A and the level of horizontal differentiation in Market B. In particular, when the price of two-sided platforms for users on one side is negative, the platform is still able to increase its gains through tying. Based on the studies of Choi and Jeon (2021) and Iacobucci and Ducci (2019), this paper applies the two-sided market theory to the economic analysis of tying and explores the mechanism of the leverage effect of tying on two-sided platforms under the relaxation of the non-negative price constraint. (2) While trying to provide a unified analytical framework for previous studies, this paper also finds that product differentiation on the buyers' side and heterogeneity on the sellers' side constitute the prerequisites for the leverage mechanism via tying on two-sided platforms, which consequently expands the theoretical study of leverage via tying in two-sided markets from the perspective of product differentiation and offers a new explanation for the construction of the two-sided platform ecological networks. (3) For competitors in the competitive products market, their profit decreases. Thus, taking the operating system market as an example, the study reveals the strategic incentives for two-sided platforms to tie in terms of consolidating their own business ecosystems and limiting competitors' access to the core areas. Thus, the paper aims to shed light on the construction of two-sided platform business ecological networks and governance of the platforms.

In the second section, we take the operating system market as an example to outline the motivation, assumption and model specification of this paper. Section 3 develops a tying model to analyze the expansion of the platform ecological network. Section 4 analyzes the equilibrium of entry and buyers' multi-homing for the analysis of the stability of the ecological network. Section 5 is the conclusion.

2. The operating system market and model specification

2.1 Background

The operating system is the first layer of software which is closest to the hardware, serving as a bridge between physical devices and resources and upper-level software application-level services. The functions of the operating system are mainly concerned with the management of five computer resources, including micro-kernel processing, memory scheduling, input and output devices, files and operations. Based on different usages, operating systems can be classified into desktop operating systems, mobile operating systems, server operating systems, cloud operating systems, etc. The majority of the desktop operating system and server operating system is Microsoft's Windows system, while the majority of a mobile operating system is Google's Android system. It is worth mentioning that Apple occupies a significant market share in both desktop and mobile operating systems.

Figure 1 shows the market share of operating systems in China, the USA, the EU and worldwide in 2020. Through comparison, we see that there are significant differences in

China compared with the USA and Europe: on the one hand, China has a higher share of the mobile OS market, revealing the boom in mobile Internet in the country, thanks to the successful low-price strategies and subscription models of local cell phone manufacturers such as Huawei and Xiaomi. On the other hand, OS X (Apple's desktop operating system) has a significantly lower market share in China than in other regions. One explanation for this is that many PC users are locked into the Windows system due to the feature of the operating system ecological network. This is similar to the EU, although the preference for Windows in China is much stronger. As for the mobile terminal's side, the main difference between China, Europe and the USA is that the version of Android in China is forked, which does not require pre-installment of applications, such as Google Search, Google Play, Google Chrome, etc. These applications often constitute an important part and profit source of the Android ecological network. The excess profits brought by the ecological network are, to a certain extent, gained by Huawei, Xiaomi and other local cell phone manufacturers who have, respectively, developed their own ecosystems on the forked version of Android. This, in turn, has contributed to the boom in mobile Internet in China.

Figure 2 depicts the market share of mobile vendors in China and other regions of the world during 2010–2021. A big difference is shown between the two regions. Since the decline

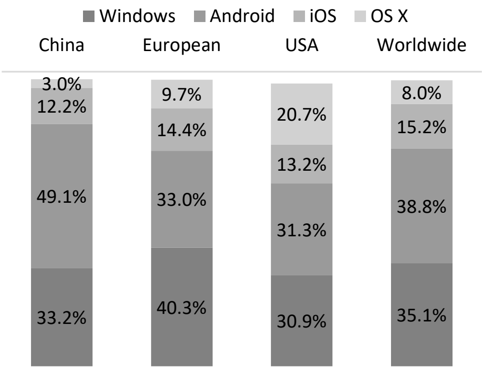
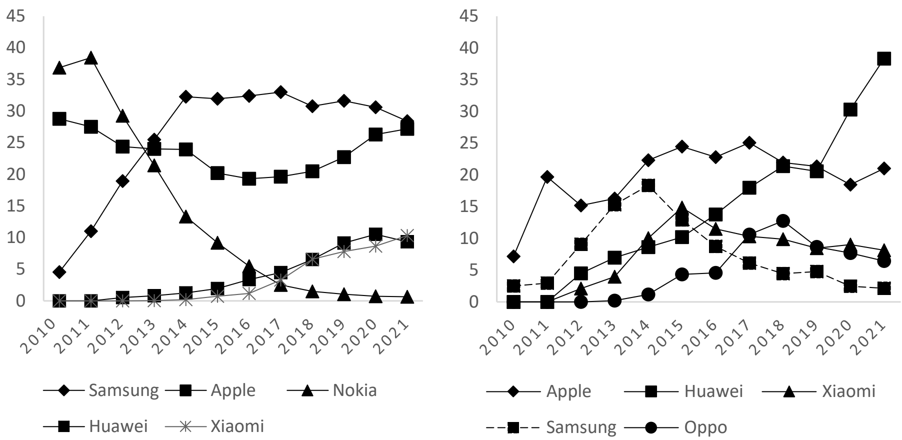


Figure 1.
Operating system market Share, 2020



Source(s): Gs.statcounter.com

Figure 2.
Mobile vendor market share in China and worldwide, 2010-2020

of Nokia, the global market has been dominated by Apple and Samsung, and the two late comers from China – Huawei and Xiaomi – are gradually occupying a certain amount of market share. The Chinese market, formerly dominated by Huawei and Apple, has seen Huawei gradually overtake Apple and become the largest cell phone manufacturer in 2019, and the trend has gone upward instead of downward with the US sanctions. This is partly due to Huawei's efforts in building a national brand and partly due to its strong capital and technological accumulation. In particular, after being added to the Entity List by the USA, Huawei undertook its endeavor to build an operating system compatible with all Android applications and all Web applications to get rid of the impact of being banned from Android by Google and tried to connect cell phones, computers, platforms, cars and other devices and officially launched the HarmonyOS in 2019. In order to cultivate its own ecosystem and gain a foothold in the operating system market where market barriers are high, Huawei began to promote HarmonyOS 2.0 as an open source. And through compatibility with Android, Huawei is able to transfer its own system applications (such as AppGallery, Huawei Browser, etc.) to HarmonyOS and expand its ecological network through free and open-source promotion. At the same time, since Google's ecosystem does not cover the Chinese market, it provides a promising environment for the establishment and development of Huawei's OS ecosystem. In order to establish itself in the operating system, Huawei needs to work with upstream and downstream firms to build the ecological network and enrich that network by pre-installing software applications, which is essentially a (mixed) bundling strategy, because as a late entrant, the tying strategy that limits product supply is not feasible.

Based on the above analysis, this paper attempts to explore in theory the mechanism of how network externalities, product/service differentiation and the tying strategy affect the success of platform operations, by taking the competition among operating systems and the entry of Huawei's operating system as examples. Further, it tries to identify the mechanism of the construction and consolidation of the ecological network of digital platforms through the analysis of entry threats and buyers' multi-homing. By doing so, it is hoped that this paper can deepen the existing relevant theoretical research while explaining the competitive strategies of firms in the digital era.

2.2 Hypotheses

To clarify the research in this paper, the following hypotheses are proposed for subsequent model specification and comparative static analysis.

- *H1.* Tying can help a two-sided platform to construct an ecological network by expanding market boundaries when the strength of network externality on the seller side is stronger enough than that of the demand side in the tied market.

Digital platforms have an incentive to tie only when the profitability of tied sales is higher than that of individual sales. Combined with the Chicago school's so-called "*SMPT*" (Single Monopoly Profit Theory), the consumer's evaluation of the primary product, the product differentiation of the tied goods and the intensity of competition in the market all affect the relative profitability (Whinston, 1990). The presence of network externalities brings changes in the value of primary products and affects the degree of competition between platforms through "*Competitive Bottlenecks*" (Armstrong, 2006; Armstrong & Wright, 2007). The symmetry of network externalities, on the other hand, affects the price structure on both sides of the platform and gives rise to a situation where platforms subsidize users on one side with users on the other. As pointed out by Song *et al.* (2018), asymmetric network externalities significantly affect the process of value creation and value capture and have important implications for the governance model of the platform. Finally, the degree of (horizontal) product differentiation directly determines the intensity of competition between the duopoly platforms, together with network externalities, and affects the strategic returns of the platform ecological network.

This assumption is necessary for that network externalities come into play with preconditions. For example, in Amelio and Jullien's (2012) and Choi and Jeon's (2021) homogeneous product-NPC frameworks, neither the strength of network externalities for primary products nor the symmetry of network externalities in the tied product market exerts an impact on the change in relative profits. The prerequisites for the mechanism at play in this paper are also tested in Appendix 4.

H2. The incentives for a digital platform to subsidize two-sided users depend mainly on the symmetry of network externalities and the intensity of product differentiation.

When there is no constraint on the platform's pricing, the asymmetry of network externalities will incentivize the platform to charge excessive prices to the highly dependent users in order to subsidize and attract users on the other side. When bundled products are faced with this decision, the platform's tying strategy is likely to lose its appeal. This is because the leverage mechanism of tying requires that the platform be able to derive additional marginal revenue from tying, which means that the price of the tied product is higher than the sum of the prices of the two types of products when sold separately (Whinston, 1990). To compensate for the loss of profit from the drop in prices of bundled products, the platform must go to the users on the other side of the tied product market. This imposes the requirement that the network externalities on the sellers' side in the tied product market should be relatively strong enough and that product differentiation be relatively weak, as is suggested in the case study of Iacobucci and Ducci (2019).

H3. Tying of digital platform can effectively deter entry into the primary product market.

The primary product market of digital platforms corresponds to the core products of the platform's ecological network. When the core products are confronted with competitive threats, digital platforms are able to integrate different products or services through tying, so that competition among single platforms is transformed into competition among the ecological networks of the platforms. Therefore, when the primary product market is facing entry threats, the tying strategy can greatly enhance the entry barrier for competitors, making it much more difficult to enter. At the same time, even if an entrant in the primary product market teams up with a competitor in the tied product market to conduct an "Envelopment" strategy, tying remains the dominant strategy for the incumbent platform, and it makes the profits of the entrant much lower than expected.

2.3 Model specification

Combining the above cases and hypotheses, we first depict the key elements of the digital market and extract them. The analysis in this paper focuses on two aspects: (1) *The construction of the ecological network of digital platform*, i.e. how digital platforms leverage market power and build their own ecological networks through tying. To clarify the net impact of the tying strategy, we assume that the two types of products are independent from each other. It can be expected that the effect of tying will be more salient when these two are complementary. Also, for the sake of analysis, we assume that the primary and tied products are single products, in the knowledge that this obviously can be extended to the scenario of multiple products. In this case, the primary product market will be designated as the core business of the platform firm, while the tied product will represent the related market it tries to capture. (2) *The stability of the digital platform ecological network* is combined with the release of Huawei's HarmonyOS to examine the presence of an entry in the primary product market (considering the decision of Entrant A2) and the buyers' multi-homing behavior also to explore the incentives for buyers to install alternative products when they have had pre-installed applications and the consequent effects. Figure 3 shows the composition of the different players in primary product, denoted by Market A, and tied product, denoted by Market B, and their interrelations, denoted by AB. The platform obtains profits by providing trading services for both buyers and sellers, and the product evaluation (net revenue) of the two types of users will be affected by the number of users on the other side, i.e. network externality. Monopolistic platform 1 provides both A and B1 products, while Platform 2 only provides differentiated product B2. The buyer chooses between buying A, buying B, buying AB and not buying at all. However, content providers decide which platform to access according to their own installation cost and the expected number of buyers on the platform under the principle of profit maximization.

In the basic model, we examine the first scenario to analyze the incentives for Platform 1 to tie and then consider the second scenario to analyze the entry and platform envelopment in the primary product market. Note that we can also view Market A dynamically as a platform ecosystem containing a number of sub-markets or products that Platform 1 integrates through tying. In this context, Platform 1 can expand its platform ecosystem by bundling different product groups together. Although potential entry into any market may pose a threat to platform competition, tying still provides Platform 1 with an incredible strategic threat, allowing it to maintain strong market power in the market with a high user base. At the same time, the entry of new competitors points the way for Platform 1's market expansion, as business models are not that difficult to learn in the digital market, especially for "*superstar*" platforms.

To simplify the analysis, we limit the analysis to two sub-markets with two-sidedness (Choi & Jeon, 2021). There are Market A and Market B which are mutually independent (e.g. operating systems and app stores), each consisting of three types of players: buyers, digital platforms and content providers (e.g. app developers, advertisers, etc.). Each platform derives revenue by providing transactional services to buyers on the demand side and content providers (CP) on the seller side. We assume that there exist two platforms (1 and 2). Platform 1 supplies products or services to both markets, and its price for users on the two sides of market $k \in \{A, B\}$ are p_1^k and q_1^k. Platform 2 only supplies products to Market B, and its price for users on the two sides are p_2^B and q_2^B. Therefore, the profit functions of Platform 1 and Platform 2 can be written as follows:

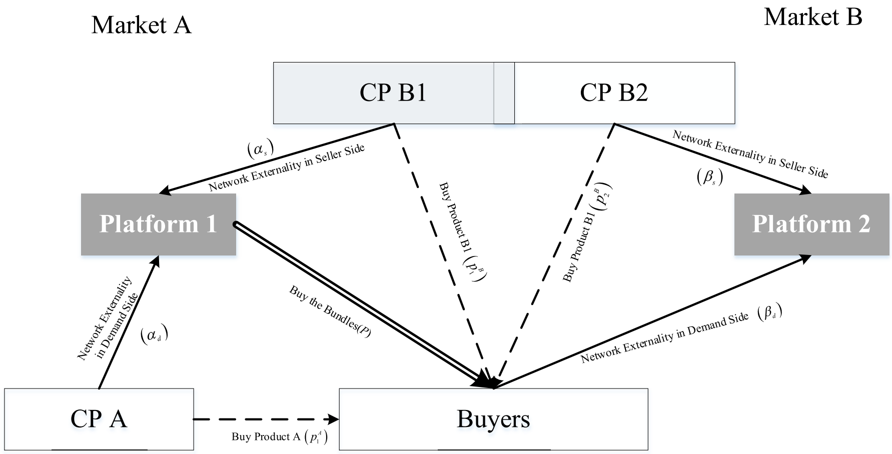


Figure 3.
The expansion of two-sided platform

$$\Pi_1 = \sum_k \pi_1^k = \sum_k \left(p_1^k d_1^k + q_1^k s_1^k\right), \ \pi_2^B = p_2^B d_2^B + q_2^B s_2^B. \tag{1}$$

where d_1^k represents the demand from buyers in market k for the products or services on Platform 1; while s_1^k represents the supply of products or services from content providers on Platform 1 in market k.

2.3.1 Primary product market model specification. In Market A, buyers and content providers trade on Platform 1. Similar to Choi and Jeon (2021), we assume that buyers' initial evaluations (u^A) about the product are heterogeneous and follow a uniform distribution on the interval [0,1]. The net utility (net benefit) of buyers and content providers are as follows:

$$u_1^A = u^A + \alpha_d s_1^A - p_1^A, \ v_i^A = \alpha_s d_i^A - q_i^A - f_i^A. \tag{2}$$

where α_d represents the strength of network externalities on the consumer (buyers) side, i.e. the incremental utility to the consumer per additional content provider. α_s represent the strength of network externalities on the content providers' (sellers) side, i.e. the increase in the content provider's profit per additional consumer. f^A represents the content provider's setup cost. We assume that the total number of content providers is 1, and that $f^A \sim U[0,1]$. In order to depict the mechanism by which tying leverages market power, we assume that Platform 1 has always held the monopoly position in Market A and there is no potential entry. The equilibrium results in the presence of potential entry will be discussed accordingly in Section 5.

2.3.2 Tied product market model specification. We assume that buyers are single-homing in Market B. Content providers can choose either single-homing or multi-homing. To conform to the actual situation, we assume that the products or services offered by the two platforms can be horizontally differentiated by buyers. Assume that Platform 1 and Platform 2 are located at each end of the Hotelling line in Market B. Assume that Platform 1 is located at Point 0 and Platform 2 is located at Point 1. Buyers are uniformly distributed on the Hotelling line. The utility function of a consumer located at x_i can be written as follows:

$$u_i^B = \overline{u}^B + \beta_d s_i^B - p_i^B - tx_i. \tag{3}$$

where $\overline{u}^B$ represents buyers' reservation utility, which we assume is great enough so that Market B is fully covered. β_d represents the network externalities on the buyers' side. t is the transport cost, i.e. the degree of horizontal differentiation in products or services offered by the two platforms. Assuming that content providers accessing the platform also incur installation costs, the net benefit (or net utility) to content providers can be written as follows:

$$v_i^B = \beta_s d_i^B - q_i^B - f_i^B. \tag{4}$$

We assume that the setup cost f_i is heterogeneous and uniformly distributed within [0,1]. For analytical purposes, we assume that buyers' evaluations of Product A and its position in Market B are independent and that two-sided users' expectations regarding each other's quantities can be realized (Katz & Shapiro, 1985). Also, to ensure that the equilibrium result of the model is unique, we assume that the following condition can be satisfied:

$$8t > (\alpha_d + \alpha_s)^2 + \beta_d^2 + 6\beta_d\beta_s + \beta_s^2. \tag{5}$$

For ease of interpretation, the economic implications of relevant variables and parameters are summarized and illustrated in Table 2.

2.3.3 Timing of the game. The timing of the game is as follows: In the first stage, Platform 1 decides whether to tie. If tying is implemented, Platform 1 offers both products in a 1:1 ratio

Table 2.
Variables and parameters

Variables	Implications
u_i^k	Net utility of buyers
$\overline{u}^i$	Initial evaluations (reserve value) of buyers
p_i^k	Price for buyer-side
P	Price for the bundles, AB
α_k	Network externality of Product A
β_k	Network externality of Product B
v_i^k	Net profit of sellers
q_i^B	Price for seller-side
t	Product differentiation
x_i	Buyer's location in Hotelling model

rather than offering one of these two separately and names the product combination as a bundle [3]. In the second stage, the two platforms simultaneously set two-sided prices (p_i^k, q_i^k) to maximize their profits. In the third stage, buyers and content providers simultaneously choose between platforms after observing the platforms' offers. We use the standard backward induction to solve the equilibrium. To this end, the paper first analyzes the choices of buyers and content providers given the implementation of tying for Platform 1 and the pricing for both platforms. Then, it solves the price under the maximized profits of the platforms in the case of (no) tying. Finally, it clarifies the incentives for Platform 1 to tie by comparing the profits of no tying and tying.

3. Expansion of platform ecological network: motivation of tying

In accordance with the method of backward induction, this paper first solves the equilibrium when the two platforms sell separately, and then solves the equilibrium when Platform 1 adopts the tying strategy. Finally, by comparing the profit difference of Platform 1 under the two equilibria, this paper illustrates the incentives for Platform 1 to tie.

3.1 No-tying

According to the fulfilled expectations hypothesis, the number of two-sided users in Market A can be obtained from formula (2–4):

$$d_A = \frac{1 - p_A - q_A\alpha_d}{1 - \alpha_d\alpha_s};\ s_A = \frac{\alpha_s(1 - p_A) - q_A}{1 - \alpha_d\alpha_s}. \tag{6}$$

Correspondingly, the respective numbers of two-sided users in Market B are:

$$d_1^B = \frac{1}{2} - \frac{p_1^B - p_2^B + \beta_d\left(q_1^B - q_2^B\right)}{2(t - \beta_d\beta_s)};\ d_2^B = \frac{1}{2} + \frac{p_1^B - p_2^B + \beta_d\left(q_1^B - q_2^B\right)}{2(t - \beta_d\beta_s)}.$$

$$s_1^B = \beta_s\left(\frac{1}{2} - \frac{p_1^B - p_2^B + \beta_d\left(q_1^B - q_2^B\right)}{2(t - \beta_d\beta_s)}\right) - q_1^B;\ s_2^B = \beta_s\left(\frac{1}{2} + \frac{p_1^B - p_2^B + \beta_d\left(q_1^B - q_2^B\right)}{2(t - \beta_d\beta_s)}\right) - q_2^B. \tag{7}$$

Substitute formula (6) and (7) into the profit functions of the platforms in formula (1), respectively. Use the first-order condition (FOC) [4] for profit maximization, and we have the equilibrium as shown in Table 3.

By comparing the effects of different parameters on the equilibrium, we obtain Proposition 1.

Proposition 1. In the absence of tying, the equilibrium result is as follows:

(1) As network externalities intensify on both sides in Market A, both buyers and content providers of Product A increase. As network externalities intensify on both sides in Market B, the number of content providers of Product B increases, while the demand of buyers who buy Product B remains constant.

(2) When $\beta_d \geq \beta_s$, the platforms' price for the users is positive, and for the content providers, negative. When $\beta_d < \beta_s$ and $(\beta_d^2 + 6\beta_d \beta_s + \beta_s^2)/8 < t \leq (\beta_s^2 + 3\beta_d\beta_s)/4$, the platforms' price for the users is negative, and the price for the content providers is positive. When $\beta_d < \beta_s$ and $t > (\beta_s^2 + 3\beta_d\beta_s)/4$, the price is positive for users on both sides.

(3) The total profit level of Platform 1 is improved with the enhancement of network externalities in Market A and the increase of traffic cost in Market B and decreases with the enhancement of network externalities in Market B. The profit level of Platform 2, on the other hand, is improved with the increase of traffic cost in Market B and decreases with the enhancement of network externality in Market B.

Proposition 1 reveals the effect of network externalities on platform pricing and equilibrium profits in the classical two-sided platforms. In Market A, Platform 1, which holds the monopoly position, is able to effectively capture consumer surplus by adjusting prices. While in Market B, the relative strength of network externalities on both sides will cause platforms to coordinate the pricing for the two sides accordingly, or even subsidize users on one side. Ultimately, the enhanced network externalities will intensify the competition between platforms.

3.2 Tying

In the presence of tying, the buyers choose between the bundled products on Platform 1 and Product B on Platform 2. Under the assumptions, buyers evaluate Products A and B independently [5], and the market is fully covered. The net utility of buyers who purchase the bundled products can be written as follows:

$$U_1 = u^A + \alpha_d s_1^A + \overline{u}^B + \beta_d s_1^{Be} - P - tx_1. \tag{8}$$

Combining the distribution characteristics of buyers, Figure 4 depicts the distribution of buyers in different scenarios: (1) In the case of stronger differentiation, the distribution of marginal buyers intersects the X-axis at $(\overline{x}_1, \overline{x}_2)$, and the following conditions are satisfied: $\overline{x}_i \in [0, 1]$, $\overline{u}_1 < 0$, $\overline{u}_2 > 1$. (2) In the case of weaker differentiation, the distribution of marginal buyers intersects the U-axis at $(\overline{u}_1, \overline{u}_2)$, under the conditions: $\overline{u}_i \in [0, 1]$, $\overline{x}_1 < 0$, $\overline{x}_2 > 1$. To simplify the analysis, we ignore the case for moderate product differentiation where the distribution of

	Market A	Market B
Price	$p_1^{A*} = \frac{2-\alpha_s(\alpha_d+\alpha_s)}{4-(\alpha_d+\alpha_s)^2}$, $q_1^{A*} = \frac{\alpha_s-\alpha_d}{4-(\alpha_d+\alpha_s)^2}$	$p_i^{B*} = \frac{4t-\beta_s^2-3\beta_d\beta_s}{4}$, $q_i^{B*} = \frac{\beta_s-\beta_d}{4}$
Quantity	$d_1^{A*} = \frac{2}{4-(\alpha_d+\alpha_s)^2}$, $s_1^{A*} = \frac{\alpha_d+\alpha_s}{4-(\alpha_d+\alpha_s)^2}$	$d_i^{B*} = \frac{1}{2}$, $s_i^{B*} = \frac{\beta_d+\beta_s}{4}$
Profit	$\pi_1^{A*} = \frac{1}{4-(\alpha_d+\alpha_s)^2}$	$\pi_i^{B*} = \frac{8t-\beta_d^2-6\beta_d\beta_s-\beta_s^2}{16}$

Table 3. Equilibrium when platform 1 does not tie

marginal buyers intersects the $(\bar{x}_1, \bar{u}_2)$ axis with the horizontal and vertical axes, respectively. In addition, considering Platform 1's dominant position in the market, we also omit the case, where $d_1 < d_2$.

According to Figure 4, the intersection of marginal buyers on the horizontal and vertical axes can be written as follows:

$$\bar{u}_1 = P - t - p_2 + q_1^A \alpha_d - d_1(\alpha_d \alpha_s + \beta_d \beta_s) + q_1 \beta_d - q_2 \beta_d + d_2 \beta_d \beta_s,$$

$$\bar{u}_2 = t + P - p_2 + q_1^A \alpha_d - d_1(\alpha_d \alpha_s + \beta_d \beta_s) + q_1 \beta_d - q_2 \beta_d + d_2 \beta_d \beta_s;$$

$$\bar{x}_1 = \frac{t + p_2 - P - q_1^A \alpha_d + d_1(\alpha_d \alpha_s + \beta_d \beta_s) - q_1 \beta_d + q_2 \beta_d - d_2 \beta_d \beta_s}{2t},$$

$$\bar{x}_2 = \frac{t + 1 + p_2 - P - q_1^A \alpha_d + d_1(\alpha_d \alpha_s + \beta_d \beta_s) - q_1 \beta_d + q_2 \beta_d - d_2 \beta_d \beta_s}{2t}. \tag{9}$$

Similar to the previous analysis, based on the distribution characteristics of buyers in Figure 4, this paper solves the equilibrium and their constraints under three different scenarios by using FOC [6] and performs the analysis to obtain Proposition 2.

Proposition 2. In the presence of tying, the equilibrium results are as follows:

(1) When $\alpha_d^2 + \beta_d^2 > \alpha_s^2 + \beta_s^2$, $P' > 0$, i.e. the price of the bundled products is positive; on the contrary, when $\alpha_d^2 + \beta_d^2 \leq \alpha_s^2 + \beta_s^2$, $P' \leq 0$, the price of the bundled products is negative, i.e. Platform 1 will subsidize the customers who buy the bundled products.

(2) When $\alpha_s > \alpha_d$, $q_1^{A'} > 0$; conversely, $q_1^{A'} \leq 0$, i.e. when the network externalities on the sellers' side are more intense than that of the buyers' side in Market A, the price of Platform 1 for content providers is positive. In the opposite scenario, Platform 1 will offer subsidies to content providers. Similarly, when $\beta_s > \beta_d$, $q_i^{B'} > 0$; conversely, $q_i^{B'} \leq 0$, i.e. when the network externalities on the sellers' side are more intense than that of the buyers' side in Market B, both platforms are inclined to charge positive prices to content providers in Market B or subsidize them if it is the other way around.

(3) $\Pi_1' > \Pi_2' > 0$, i.e. the profits of both platforms in the equilibrium are non-negative and the profit level of Platform 1 is always higher than that of Platform 2.

The proof can be found in Appendix 2. Proposition 2 shows that the relative strength of the network externalities on both sides of the platform is the key element that determines whether the platform charges or subsidizes them. Meanwhile, the tying strategy of Platform 1 links the two markets, making the price of the bundled products susceptible to the relative

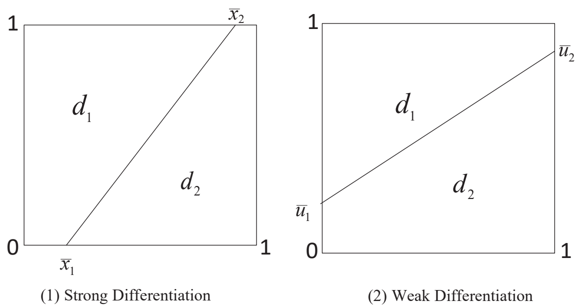


Figure 4.
Buyers demand distribution

strength of network externalities in both markets; while on the sellers' side, in the absence of tying, the price of the platform for content providers depends largely on the relative strength of the network externalities on both sides of the platform in the market in which it operates. At the same time, in the case of horizontal difference, due to its monopoly position in Market A, Platform 1 can always obtain higher profits than Platform 2, thus revealing the importance of tying in the process of constructing the firm's own business ecosystem. By integrating their own products with core advantages with non-advantageous or old resources, the firms can improve the cohesiveness of their business ecosystems and increase brand awareness.

3.3 Analysis of the incentives for Platform 1 to tie

With the equilibrium from sales of separate and tied items on Platform 1 examined in the above sections, in this section, we will examine the incentives for Platform 1 to tie based on three scenarios of tied sales. We can summarize the conclusions from the following two aspects through deduction. In the former case, since β_s is much greater than β_d, the price of Platform 1 for the bundled items is lower than the sum of the prices of the corresponding two types of products in the benchmark mode, and thus the relative price is negative. At this point, Platform 1 can obtain a higher profit through tying relative to the benchmark model. In the latter case, when the strength of the network externalities on the sellers' side is weak in both markets, Platform 1 will charge a price for the bundled products that are lower than the sum of the prices of the corresponding products in the benchmark model. Meanwhile, when the strength of the network externalities on the sellers' side is relatively weak in Market A and the strength of the network externalities on the sellers' side is relatively strong in Market B, Platform 1 can obtain a higher profit level than under the benchmark model, suggesting that Platform 1 has an incentive to tie at this point.

Combining the above analysis and Propositions 1 and 2, we have Proposition 3.

Proposition 3. In the equilibrium of tying, compared to the benchmark model, the results are as follows:

(1) When t is greater and β_s is greater, or when t is lesser and both α_j and β_j are lesser, $\Delta P < 0$; conversely, $\Delta P \geq 0$, i.e. the price of the bundled items is lower than the sum of the corresponding product prices in the benchmark model; if it is the other way around, Platform 1 may charge a higher price than the sum of the two product prices in the benchmark model. When $t \in [t_1, t_2]$, if $d_1^{AB'} \geq d_1^{A^*}$ is satisfied, then Platform 1 will have an inventive to tie.

(2) When $\alpha_s > \alpha_d$, $\Delta q_1^A > 0$; conversely, $\Delta q_1^A \leq 0$. When $\beta_s > \beta_d$, $\Delta q_1^B > 0$ and $\Delta q_2^B < 0$; conversely, $\Delta q_1^B \leq 0$ and $\Delta q_2^B \geq 0$, that is, the price offered by the two platforms to content providers in Market B also depends mainly on the relative strength of network externalities on both sides, and the price offered by the two platforms to content providers varies to the same extent and in opposite directions.

(3) When α_j and M are lesser and $\beta_s > \beta_d$, $\Pi_1' > \Pi_1^*$; conversely, $\Pi_1' \leq \Pi_1^*$. That is, when the network externalities of Market A are strong, and the network externalities of Market B are relatively weak and demonstrate a more obvious asymmetry, the profit of Platform 1 will be higher than the corresponding profit level in the benchmark model. When the horizontal differentiation of Market B is moderate and the externality on the buyers' side is strong, the profit of Platform 2 may be higher than the corresponding level when sold separately; conversely, the profit of Platform 2 will decrease.

The proof can be found in Appendix 3. Proposition 3 reveals the important economic implication that, the incentives for two-sided platforms to tie are mainly influenced by the strength of network externalities on both sides in Market A and the horizontal differentiation in Market B. When the total network externality in Market A is weak and the horizontal differentiation in Market B is strong, Platform 1 is able to increase its profits through tying, which can help moderate market competition, thus allowing Platform 2 to also gain a relatively high profit. The main reason behind this is that, the increased network effect on the sellers' side in Market B helps to attract more access from multi-homing sellers, thus expanding the market demand. In addition, Proposition 3 reveals the leverage mechanism of tying on two-sided platforms under the analytical framework of this paper. To identify the differences from Choi and Jeon (2021), this paper further explores the mechanism of leverage theory of tying when considering product differentiation strategy in three aspects, using a simplified model on the differentiation of buyers and content providers, as shown in Corollary 1.

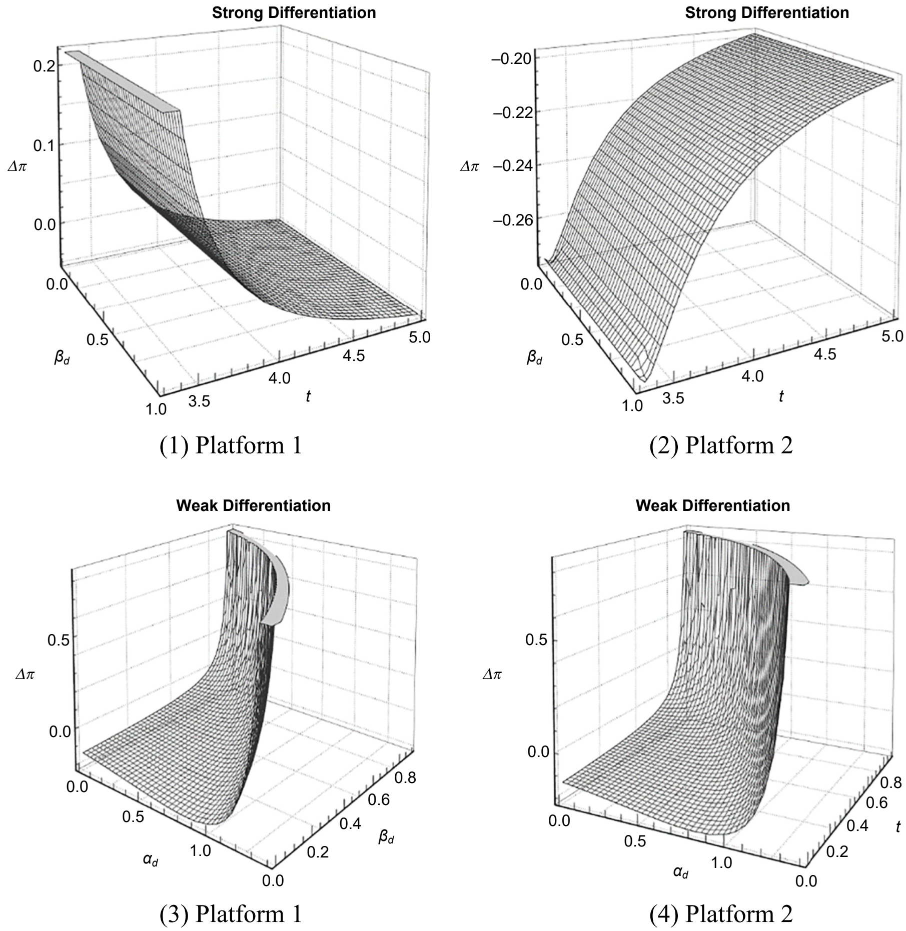
Corollary 1. Platform 1 has an incentive to tie only if buyers are horizontally differentiated and content providers are heterogeneous in Market B, and the condition of equilibrium is not affected by the NPC.

The proof can be found in Appendix 4. Corollary 1 shows that product differentiation is a necessary condition for platforms to tie, and the conclusion equally applies to subsidies that prevail in reality. In particular, for the operating system market, tying can help firms to consolidate their business ecosystems and implement market segmentation and user value reengineering through the product differentiation strategy. What role, then, does tying play when the core business of the firms' ecosystems face challenges? In the following section, we analyze the potential entry problems in the primary product market to further explore the stability of the business ecosystems. Intuitively, Figure 5 depicts the profit change of platforms' tying with strong and weak differentiation. It is shown that there are uncertain changes by tying of Platform 1 and highlights the key effect of the relative strength of network externalities and product differentiation on its incentive to tie, while Platform 2's profit is definitely decreased. This suggests that tying can further strengthen a monopolistic platform's network advantage and undercut a competitor's market position, although not drive it out of the market.

This conclusion has important management significance. In terms of operating systems, this undoubtedly raises the entry barrier for new entrants, forcing them to explore more competitive products to break the blockade of incumbents, such as browsers, security software, search engines, etc. With the continuous expansion of the platform ecology, this difficulty will undoubtedly accumulate. In order to overcome this limitation, we can consider the loudness intensity of bilateral network effects and platform service differences to find a breakthrough. Therefore, the competitive disadvantaged platform can strengthen the network externality intensity of the buyer side, improve the level of platform service differentiation and weaken the tie-in motivation of the monopoly platform.

4. Multi-homing buyers

Although operating system firms can coerce cell phone manufacturers to pre-install their own add-on software in various ways, still, buyers can find a way to download their preferred alternative software when using it. From the modeling perspective, this suggests the multi-homing of buyers' use of the tied Product B. Since multi-homing for content providers has been discussed in the benchmark model, in this part, we will analyze buyers' multi-homing to further explore the stability of the ecological network of digital platforms. Similar to the study of Armstrong and Wright (2007), in a modelling framework *a la* Hotelling model, buyers with similar net benefits from the two types of tied products (B1 and B2) have an incentive to multi-home, provided that multi-homing can generate sufficient additional benefits. It can be proven that under this Hotelling model with setup costs, buyers will choose to multi-home if



Note(s): In the case of strong differentiation, parameter assignments satisfy: $t = 3$, $\beta_d = 0.5$, $\beta_s = 3.4$ in strong differentiation. t reflects the strength of platform service differentiation relative to network externalities, while α_s and β_s reflects the influence of the relative strength of network externalities between both sides of platform. Parameter assignments $\alpha_d = 0.5$, $\beta_d = 0.5$, $t = 0.45$ in weak differentiation

Figure 5.
Profit changes of two-sided platforms

their reservation value of the tied products is high enough. To better describe the impact of buyers multi-homing, we consider the interior solution for the co-existence of single-homing and multi-homing buyers, i.e. $d_1 + d_2 > 1$.

Different from the previous analysis, in the presence of two-sided multi-homing, consumer attractiveness decreases, which weakens the incentive for Platform 1 to leverage market power to the content providers' side in the tied product market. Therefore, Platform 1 will be less willing to subsidize the bundled items. Figure 6 shows the variation of the incumbent platform's profit compared to when sold alone when the parameters constraints are given. We can see that the parameter interval for Platform 1 to tie is mainly influenced by the

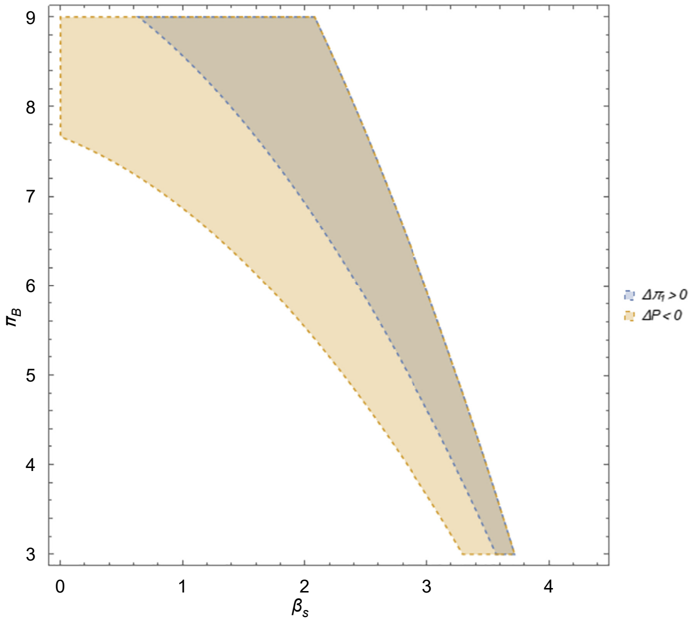


Figure 6. The Monopolist's profit with multi-homing buyers

Note(s): The parameter assignment: $\alpha_d = 0.5$, $\alpha_s = 1$, $t = 8$, $\beta_d = 1$

attractiveness of competitors ($\overline{u}_B$) and the strength of network externalities of the sellers of the tied products. In particular, when $\overline{u}_B$ is so low that buyers are not willing to home more than two platforms, the parameter interval in which Platform 1 has an incentive to tie will be lower, thus supporting our inference.

5. Conclusion

With the rapid development of the digital economy and the continuous evolution of firms' business ecosystems, an increasing number of platforms have attempted to leverage market power and consolidate their business ecosystems through tying. This paper uses a dynamic game model to explore the mechanism by which the leverage effect of tying works in two-sided markets and its impact on market competition. The results show that (1) the price offered by two-sided platforms to users on both sides depends on the relative strength of network externalities of users on the two sides in Market B: platforms tend to charge relatively higher prices to users with stronger network externalities and relatively lower or even negative prices to users on the other side. (2) The incentives for two-sided platforms to tie are mainly influenced by the strength of network externalities on both sides in Market A and the horizontal differentiation in Market B: when the total network externality in Market A is weak and the horizontal differentiation in Market B is strong, Platform 1 can increase its profits by tying and help make the market competition less intense, so that Platform 2 also gains relatively higher profit. Thus, when two-sided platforms offer negative pricing for the users (e.g. buyers) on one of the two sides, the platforms can still earn higher profits through tying. Therefore, this paper extends the existing research by relaxing the "*non-negative price constraint*." (3) When there is a potential entry in the primary product market, tying can mitigate the loss caused by entry through market segmentation.

The research in this paper theoretically expands and furthers the studies of Choi and Jeon (2021) and Iacobucci and Ducci (2019) with important practical implications. At the same time, this paper emphasizes that in the digital era characterized mainly by two-sided markets, the incentives for firms to tie are closely related to their product differentiation strategies and the stability of their business ecosystems. Therefore, relying solely on technological innovation cannot help highly efficient firms to enter the market in an efficient way. Only when product innovation and sales strategies are combined, different resources and multiple layouts integrated, can they effectively enter the market. Regarding the operating system market, Huawei, as a potential highly efficient entrant, should not try to seize the market simply by relying on its advantages at the system level but should accumulate its advantages in the browser market, search engine market, application store market, etc., through R&D or integration of other resources, so as to effectively overcome the restriction of Android's inherent ecosystem and successfully integrate into the market. Although we have studied the formation and expansion mechanism of platform ecology, there are still some aspects that need to be further investigated to explore the market boundaries of platform ecological expansion, such as platform ecology competition and cooperation strategy, product innovation strategy and platform merge strategy. At the same time, empirical studies similar to Kim, Peter, Liad, and Ran (2022) and McCalman (2022) are also worth exploring.

Notes

1. Pure bundling is closely related to tying. Bundling refers to the act of firms selling different products as a package. Depending on whether different products are sold separately, bundling can be divided into pure bundling and mixed bundling. Pure bundling means that the composition ratio of different products is fixed, while tied sales allow different quantities of tied products to be sold together with the primary product. Therefore, it is generally considered that pure bundling is a special case of tying, but economists often do not distinguish between the two, and the analysis in this paper follows this practice.

2. Belleflamme (2005) has proved that, a sufficient condition for a monopoly to make the tying profitable is for the correlation coefficient between the two products to be below 0.38; while Armstrong (2013) demonstrates in a more generalized framework that firms have an incentive to bundle when the elasticity of demand for the bundled product is higher than that of the component product.

3. The analysis in this paper also applies to the case where Platform 1 simultaneously offers product A. For example, when Products A and B are complementary. Intuitively, with the ability to better coordinate the prices of Products A and B_1, Platform 1 has an incentive to tie and reap excess profits by raising the asking price of content providers of Product A and in Market B.

4. The second-order condition (SOC) for maximizing the profit of two-sided platforms in Market A is $4 > (\alpha_d + \alpha_s)^2$; The SOC in market B is $8t > \beta_d^2 + 6\beta_d\beta_s + \beta_s^2$. From formula (5), the above conditions are naturally satisfied.

5. Corresponding to the antitrust case against Google's Android, the large amount of software in the app store (Google Play) makes it possible for buyers to choose between, for example, mobile browsers and search engines, without their evaluation of the app store being significantly influenced by the latter's product differentiation. This assumption allows this paper to avoid the complex situation as a result of consumer evaluation of Product A and the differences of Product B.

6. Since the analytical solution for the moderately differentiated case cannot be obtained, this paper applies the general analysis to give the conditions for profit enhancement by tying and supports them with numerical examples, see Appendix 2.

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(The Appendix follows overleaf)

Appendix 1

Study	Theme	Pre-conditions	Key findings
Bowman (1957)	Leverage theory of tying	Homogeneity of consumers and products	Leverage theory is not the reason for tying, i.e. The Chicago School Critique
Whinston (1990), Mathewson and Winter (1997)		Economies of scale, entry uncertainty, quality differentiation, etc.	The Chicago School Critique fails under certain conditions
Bakos and Brynjolfsson (1999)	Tying for Product differentiation	Bundling sufficient varieties of goods to increase the "predictive value" above their marginal cost	Bundling large numbers of unrelated information goods can be surprisingly profitable
Chen (1997)		Simultaneous action, homogeneous products and complete information	Bundling is an equilibrium strategy of one or both of the duopolists for its role as a product-differentiation device
Chao and Derdenger (2013)		Sufficient proportion of installed base and two-sidedness	Mixed bundling acts as a price discrimination tool segmenting the market more efficiently
Rochet and Tirole (2008)	Tying with network externalities	Competing platforms, informed buyers and homogenous merchants	Tying provides a strategy to balance the two-sided pricing in two-sided platforms, raising both platform benefits and social welfare
Choi (2010)		Exogenous amount of exclusive content on the seller-side	Tying can boost market demand by increasing the proportion of multi-homing buyers, enhancing platform profit and social welfare
Mariotto and Verdier (2015)		Firms' incentive to tie depend mainly on buyers' acceptance of the primary product	Price parity(tying) reduces the total transaction fee paid by buyers and merchants for making a transaction on the platform
Iacobucci and Ducci (2019)		Homogeneous buyers, multi-homing sellers	Google can attract additional advertisers on its vertical search platform that would have possibly advertised on competing vertical search platforms without a tie
Choi and Jeon (2021)		NPC, homogeneous buyers	Tying provides a mechanism to circumvent the constraint in the tied market without inviting aggressive responses by the rival firm

Note(s): We do not make a clear distinction between tying and bundling, though some scholars believe they are quite different

Table A1. Summary of studies about firms' motivations of tying

Appendix 2

2.1. Equilibrium prices and profits in the case of strong differentiation

Combining the expectations hypothesis and formula (9), the number of consumers and content providers can be calculated as follows:

$$d_1^B = \frac{1}{2}\left(1 + \frac{1 - 2P + 2p_2^B + \alpha_d\left(\alpha_s - 2q_1^A\right) + 2\beta_d\left(q_2^B - q_1^B\right)}{2t - \alpha_d\alpha_s - 2\beta_d\beta_s}\right),$$

$$d_2^B = \frac{1}{2}\left(1 - \frac{1 - 2P + 2p_2^B + \alpha_d\left(\alpha_s - 2q_1^A\right) + 2\beta_d\left(q_2^B - q_1^B\right)}{2t - \alpha_d\alpha_s - 2\beta_d\beta_s}\right);$$

$$s_1^B = \frac{\beta_s}{2}\left(1 + \frac{1 - 2P + 2p_2^B + \alpha_d\left(\alpha_s - 2q_1^A\right) + 2\beta_d\left(q_2^B - q_1^B\right)}{2t - \alpha_d\alpha_s - 2\beta_d\beta_s}\right) - q_1^B,$$

$$s_2^B = \frac{\beta_s}{2}\left(1 - \frac{1 - 2P + 2p_2^B + \alpha_d\left(\alpha_s - 2q_1^A\right) + 2\beta_d\left(q_2^B - q_1^B\right)}{2t - \alpha_d\alpha_s - 2\beta_d\beta_s}\right) - q_2^B. \quad \text{(A1)}$$

From formula (9), it is suggested that the number of consumers and content providers on platform i is decreasing with respect to the price of the product they purchase, while the price is increasing with respect to another platform. This finding is identical to the nature of the demand function in traditional competitive markets. Meanwhile, the number of two-sided users on Platform 1 decreases as transport cost increases, while the number of two-sided users on Platform 2 increases, reflecting the influence of the degree of product differentiation on the effect of tying on Platform 1. Intuitively, Platform 1 gains a competitive advantage through tying, and the increase in transport cost moderates the competition between platforms. Therefore, the increase in transport cost diminishes this competitive advantage of Platform 1. The increased network externalities moderate the effect of increased transport costs on platform competition to some extent, thus making the competition between platforms more intense.

Substitute (A1) into the profit functions of the two platforms, and use the FOC for profit maximization, we have:

$$P' = \frac{\left(4t - \alpha_d\alpha_s - \alpha_s^2 - 3\beta_d\beta_s - \beta_s^2\right)\left(1 + M - 2\alpha_d\alpha_s\right)}{2\left[2M - \left(\alpha_d^2 + 4\alpha_d\alpha_s + \alpha_s^2\right)\right]},$$

$$p_2^{B'} = \frac{\left[M - 1 - \left(\alpha_d + \alpha_s\right)^2\right]\left(4t - 2\alpha_d\alpha_s - 3\beta_d\beta_s - \beta_s^2\right)}{2\left[2M - \left(\alpha_d^2 + 4\alpha_d\alpha_s + \alpha_s^2\right)\right]};$$

$$q_1^{B'} = \frac{\left(\beta_s - \beta_d\right)\left(1 + M - 2\alpha_d\alpha_s\right)}{2\left[2M - \left(\alpha_d^2 + 4\alpha_d\alpha_s + \alpha_s^2\right)\right]},\ q_1^{A'} = \frac{\left(\alpha_s - \alpha_d\right)\left(1 + M - 2\alpha_d\alpha_s\right)}{2\left[2M - \left(\alpha_d^2 + 4\alpha_d\alpha_s + \alpha_s^2\right)\right]};$$

$$q_2^{B'} = \frac{\left(\beta_s - \beta_d\right)\left[M - 1 - \left(\alpha_d + \alpha_s\right)^2\right]}{2\left[2M - \left(\alpha_d^2 + 4\alpha_d\alpha_s + \alpha_s^2\right)\right]}. \quad \text{(A2)}$$

where the superscript '$\prime$' represents the equilibrium results in the presence of tying, $M \equiv 6t - \left(\beta_d^2 + 4\beta_d\beta_s + \beta_s^2\right)$. The equilibrium price of the platforms reveals the impact of network externalities on the price structure: (1) combining formula (1) and comparing the transport cost, we find that when $\alpha_d^2 + \beta_d^2 \geq \alpha_s^2 + \beta_s^2$, $P' \geq 0$; when $\alpha_d^2 + \beta_d^2 < \alpha_s^2 + \beta_s^2$ and $t < \left(\alpha_d\alpha_s - \alpha_s^2 - 3\beta_d\beta_s - \beta_s^2\right)/4$, $P' < 0$; when $t \geq \left(\alpha_d\alpha_s + \alpha_s^2 + 3\beta_d\beta_s + \beta_s^2\right)/4$, $P' \geq 0$. This demonstrates that whether the price of the bundles is positive is determined by the relative impact of transport cost and network externalities on both sides of the platform, as well as the relative strength of network externalities between both sides. (2) The equilibrium price of the two platforms for content providers shows that whether the price is positive or negative is determined by the relative strength of network externalities: when the network externalities on the sellers' side are relatively stronger than on the buyers' side, the platform is inclined to charge positive prices or hopes to subsidize them if it's the other way around. (3) From the price of Platform 2 to consumers, we have: when $(\beta_d^2+6\beta_d\beta_s+\beta_s^2-1)(\beta_d-\beta_s)\geq 0$, $p_2^{B'}\geq 0$; conversely, when the transport cost is relatively low, $p_2^{B'}<0$; when the transport cost is relatively high, $p_2^{B'}\geq 0$. The above results reflect the impact of the relative strength of network externalities on the price structure (Armstrong, 2006).

Substitute the above formula into formula (A2), the number of two-sided users in equilibrium can be obtained:

$$d_1^{B'}=\frac{1}{2}\left(1+\frac{2+\alpha_d^2+\alpha_s^2}{2M-(\alpha_d^2+4\alpha_d\alpha_s+\alpha_s^2)}\right),\ d_2^{B'}=\frac{1}{2}\left(1-\frac{2+\alpha_d^2+\alpha_s^2}{2M-(\alpha_d^2+4\alpha_d\alpha_s+\alpha_s^2)}\right);$$
$$s_1^{A'}=\frac{(\alpha_d+\alpha_s)(1+M-2\alpha_d\alpha_s)}{2\left[2M-(\alpha_d^2+4\alpha_d\alpha_s+\alpha_s^2)\right]},\ s_1^{B'}=\frac{(\beta_d+\beta_s)(1+M-2\alpha_d\alpha_s)}{2\left[2M-(\alpha_d^2+4\alpha_d\alpha_s+\alpha_s^2)\right]},$$
$$s_2^{B'}=\frac{(\beta_d+\beta_s)\left[M-1-(\alpha_d+\alpha_s)^2\right]}{2\left[2M-(\alpha_d^2+4\alpha_d\alpha_s+\alpha_s^2)\right]}. \tag{A3}$$

Formula (A3) shows the extent of influence of transport cost and network externalities on the number of two-sided users. With the increase in transport cost (or the decrease of network externalities), Platform 1 loses its advantage of scale. In particular, when the strength of network externalities on both sides of Market A approaches 0, Market A degenerates to one-sided and Platform 1 gains additional consumers ($1/M$) and content providers ($(\beta_d+\beta_s)/2M$) through tying. Besides, from $d_i'\in[0,1]$, $s_i'\in[0,1]$, we have $M\geq 1+(\alpha_d+\alpha_s)^2$.

Further, by substituting the equilibrium price and the number of two-sided users into the profit functions, the profits of the platforms can be obtained as follows:

$$\Pi_1'=\frac{\left[M+2t-(\alpha_d+\alpha_s)^2-2\beta_d\beta_s\right](M+1-2\alpha_d\alpha_s)^2}{4\left[2M-(\alpha_d^2+4\alpha_d\alpha_s+\alpha_s^2)\right]^2};$$
$$\pi_2'=\frac{(M+2t-4\alpha_d\alpha_s-2\beta_d\beta_s)\left[M-1-(\alpha_d+\alpha_s)^2\right]^2}{4\left[2M-(\alpha_d^2+4\alpha_d\alpha_s+\alpha_s^2)\right]^2}. \tag{A4}$$

Formula (A4) shows that the profit level of both platforms increases with the rise of transport cost and decreases with the enhancement of network externalities on both sides of the platforms. At the same time, by comparing the equilibrium profits of the two platforms, it is clear that the profit level of Platform 1 when implementing the tying strategy is always higher than that of Platform 2. This suggests that tying can help Platform 1 to leverage its monopoly power from Market A to Market B. In particular, Platform 2 may be excluded from the market if it requires higher costs to operate in Market B.

Substitute the above equilibrium results into the constraints where $\bar{x}_i\in[0,1]$, $\bar{u}_1<0$ and $\bar{u}_2>1$ in the case of low level of differentiation, we have the following relation:

$$t>t_1\equiv\frac{4+(\alpha_d+\alpha_s)^2+\beta_d^2+4\beta_d\beta_s+\beta_s^2+\Delta}{12} \tag{A5}$$

$\Delta\equiv\{15-6(\beta_d^2+4\beta_d\beta_s+\beta_s^2)+[1+(\alpha_d+\alpha_s)^2+\beta_d^2+4\beta_d\beta_s+\beta_s^2]^2\}^{1/2}$. Formula (A5) shows that when transport cost is relatively high, the point of consumers who have no differentiation between Platform 1 and Platform 2 intersects with x.

Combing the above analysis, we have *lemma* 1.

Lemma 1. In the equilibrium where the level of differentiation is relatively weak, ($t\geq t_1$), the implementation of tying on Platform 1 leads to the following results:

(1) When $\alpha_d^2+\beta_d^2\geq\alpha_s^2+\beta_s^2$ or $t\geq(\alpha_d\alpha_s+\alpha_s^2+3\beta_d\beta_s+\beta_s^2)/4$, $P'\geq 0$; when $\alpha_d^2+\beta_d^2<\alpha_s^2+\beta_s^2$ and $t<(\alpha_d\alpha_s-\alpha_s^2-3\beta_d\beta_s-\beta_s^2)/4$, $P'<0$, i.e. when the network externalities on the buyers' side of the two markets are stronger than on the sellers' side or when the two platforms in Market B are sufficiently differentiated, Platform 1's price for the bundles is positive. Conversely, Platform 1 may charge negative prices for the bundles, i.e. subsidize the consumers who purchase the bundles. In addition, when $(\beta_d^2+6\beta_d\beta_s+\beta_s^2-1)(\beta_d-\beta_s)\geq 0$, $p_2^{B'}\geq 0$; conversely, when the transport cost is relatively low, $p_2^{B'}<0$, suggesting that Platform 2's price to consumers is influenced by the strength of the network externalities and the symmetry on both sides.

(2) When $\alpha_s>\alpha_d$, $q_1^{A'}>0$; conversely, $q_1^{A'}\leq 0$, i.e. when the network externalities on the buyers' side are stronger than on the sellers' side in Market A, Platform 1's price to content providers is positive; and if its the other way around, Platform 1 will subsidize them. In the same vein, when $\beta_s>\beta_d$, $q_i^{B'}>0$; conversely, $q_i^{B'}\leq 0$, i.e. when the network externalities on the sellers' side are relatively stronger than on the buyers' side in Market B, both platforms tend to charge positive prices to content providers in Market B and subsidize them in the opposite situation.

(3) $\Pi_1'>\Pi_2'>0$, i.e. the profits of the two platforms in equilibrium are non-negative and the profit level of Platform 1 is always higher than that of Platform 2.

Lemma 1 shows that Platform 1's price for bundles depends on the relative strength of the network externalities on both sides of the two market platforms ($\alpha_d^2+\beta_d^2\geq\alpha_s^2+\beta_s^2$); while Platform 2's price for consumers depends mainly on the strength of the network externalities on both sides in Market B and the degree of symmetry. This reflects that in the case of weak differentiation, Platform 1 flexibly adjusts the prices on both sides in accordance with the relative strength of the network externalities of two-sided users in the two markets; while Platform 2 focuses more on the strength of the network externalities and the symmetry in Market B and adjusts its prices appropriately. Ultimately, by coordinating the two-sided prices in the two markets, Platform 1 is able to obtain a higher profit than Platform 2.

2.2. Equilibrium prices and profits in the case of weak differentiation

Similar to the previous analysis, in the case of strong differentiation, the number of consumers of Platform 1 and Platform 2 is represented by the upper and lower "trapezoidal" areas in Figure 2 (2), respectively, where $d_2=(u_1+u_2)/2$, $d_1=1-d_2$. Similarly, combining formula (9) and the achievable expectations hypothesis, the number of consumers and content providers can be calculated as follows:

$$\begin{aligned}
d_1^{AB}&=\frac{1-\beta_d\beta_s}{1-\alpha_d\alpha_s-2\beta_d\beta_s}-\frac{P-p_2^B+q_1^A\alpha_d-\beta_d\left(q_2^B-q_1^B\right)}{1-\alpha_d\alpha_s-2\beta_d\beta_s},\\
d_2^{A}&=\frac{P-p_2^B+q_1^A\alpha_d-\beta_d\left(q_2^B-q_1^B\right)}{1-\alpha_d\alpha_s-2\beta_d\beta_s}-\frac{\alpha_d\alpha_s+\beta_d\beta_s}{1-\alpha_d\alpha_s-2\beta_d\beta_s};\\
s_1^B&=\beta_s\left(\frac{1-\beta_d\beta_s}{1-\alpha_d\alpha_s-2\beta_d\beta_s}-\frac{P-p_2^B+q_1^A\alpha_d-\beta_d\left(q_2^B-q_1^B\right)}{1-\alpha_d\alpha_s-2\beta_d\beta_s}\right)-q_1^B,\\
s_2^B&=\beta_s\left(\frac{P-p_2^B+q_1^A\alpha_d-\beta_d\left(q_2^B-q_1^B\right)}{1-\alpha_d\alpha_s-2\beta_d\beta_s}-\frac{\alpha_d\alpha_s+\beta_d\beta_s}{1-\alpha_d\alpha_s-2\beta_d\beta_s}\right)-q_2.
\end{aligned}\tag{A6}$$

Similar to the case of weak differentiation, by substituting formula (A6) into the profit functions of the platforms, and using the FOC for profit maximization, we have the following relation:

$$P'=\frac{\left(2-\alpha_d\alpha_s-\alpha_s^2-3\beta_d\beta_s-\beta_s^2\right)\left(4-2\alpha_d\alpha_s-\beta_d^2-4\beta_d\beta_s-\beta_s^2\right)}{2\left[2\left(3-\beta_d^2-4\beta_d\beta_s-\beta_s^2\right)-\left(\alpha_d^2+4\alpha_d\alpha_s+\alpha_s^2\right)\right]},$$

$$q_1^{B'} = \frac{(\beta_s - \beta_d)(4 - 2\alpha_d\alpha_s - \beta_d^2 - 4\beta_d\beta_s - \beta_s^2)}{2[2(3 - \beta_d^2 - 4\beta_d\beta_s - \beta_s^2) - (\alpha_d^2 + 4\alpha_d\alpha_s + \alpha_s^2)]},$$
$$q_1^{A'} = \frac{(\alpha_s - \alpha_d)(4 - 2\alpha_d\alpha_s - \beta_d^2 - 4\beta_d\beta_s - \beta_s^2)}{2[2(3 - \beta_d^2 - 4\beta_d\beta_s - \beta_s^2) - (\alpha_d^2 + 4\alpha_d\alpha_s + \alpha_s^2)]},$$
$$q_2^{B'} = \frac{(\beta_s - \beta_d)\left[2 - (\alpha_d + \alpha_s)^2 - \beta_d^2 - 4\beta_d\beta_s - \beta_s^2\right]}{2[2(3 - \beta_d^2 - 4\beta_d\beta_s - \beta_s^2) - (\alpha_d^2 + 4\alpha_d\alpha_s + \alpha_s^2)]};$$
$$p_2^{B'} = \frac{(2 - 2\alpha_d\alpha_s - 3\beta_d\beta_s - \beta_s^2)\left[2 - (\alpha_d + \alpha_s)^2 - \beta_d^2 - 4\beta_d\beta_s - \beta_s^2\right]}{2[2(3 - \beta_d^2 - 4\beta_d\beta_s - \beta_s^2) - (\alpha_d^2 + 4\alpha_d\alpha_s + \alpha_s^2)]}. \tag{A7}$$

The effect of the relative strength of the network externalities on both sides of the platforms on the price is calculated by using the inequality condition of formula (1): (1) the price of the bundles is non-negative when $\alpha_d^2 + \beta_d^2 \geq \alpha_s^2 + \beta_s^2$; conversely, the price of the bundles may be negative when the network externalities are strong. Intuitively, the tying strategy of Platform 1 links Market A and Market B, such that whether the price that Platform 1 charges for the bundles is positive or negative is simultaneously determined by the relative strength of the network externalities on both sides of the platform in the two markets. (2) Platform 2 has higher volatility in the price to consumers. When the network externalities are relatively weak, or when the network externalities are strong and $\alpha_d^2 + \beta_d^2 \leq \alpha_s^2 + \beta_s^2$, the price of Platform 2 is positive; on the contrary, when the network externalities are moderately strong, the price of Platform 2 may be negative, i.e. it subsidizes the consumers. (3) The platform's price to content providers depends mainly on the relative strength of the network externalities on both sides of the platform in the market in which it is located. When the network externalities on the sellers' side are relatively stronger, the platform's price to content providers is positive; conversely, the platform may subsidize them.

Further, by substituting formula (A7) into formula (A6), we have the following relation:

$$d_1^{AB'} = \frac{1}{2} + \frac{2 + \alpha_d^2 + \alpha_s^2}{2[2(3 - \beta_d^2 - 4\beta_d\beta_s - \beta_s^2) - (\alpha_d^2 + 4\alpha_d\alpha_s + \alpha_s^2)]},$$
$$s_1^{A'} = \frac{(\alpha_d + \alpha_s)(4 - 2\alpha_d\alpha_s - \beta_d^2 - 4\beta_d\beta_s - \beta_s^2)}{2[2(3 - \beta_d^2 - 4\beta_d\beta_s - \beta_s^2) - (\alpha_d^2 + 4\alpha_d\alpha_s + \alpha_s^2)]};$$
$$s_1^{B'} = \frac{(\beta_d + \beta_s)(4 - 2\alpha_d\alpha_s - \beta_d^2 - 4\beta_d\beta_s - \beta_s^2)}{2[2(3 - \beta_d^2 - 4\beta_d\beta_s - \beta_s^2) - (\alpha_d^2 + 4\alpha_d\alpha_s + \alpha_s^2)]};$$
$$d_2^{B'} = \frac{1}{2} - \frac{2 + \alpha_d^2 + \alpha_s^2}{2[2(3 - \beta_d^2 - 4\beta_d\beta_s - \beta_s^2) - (\alpha_d^2 + 4\alpha_d\alpha_s + \alpha_s^2)]},$$
$$s_2^{B'} = \frac{(\beta_d + \beta_s)\left[2 - (\alpha_d + \alpha_s)^2 - \beta_d^2 - 4\beta_d\beta_s - \beta_s^2\right]}{2[2(3 - \beta_d^2 - 4\beta_d\beta_s - \beta_s^2) - (\alpha_d^2 + 4\alpha_d\alpha_s + \alpha_s^2)]}. \tag{A8}$$

It can be seen from formula (A8) that the tying strategy of Platform 1 helps it obtain a relatively higher market share, and the number of consumers of the bundles increases with the enhancement of the network externalities.

In addition, with $d_i \in [0, 1]$ and $s_i \in [0, 1]$, it can be obtained that

$$2 \geq (\alpha_d + \alpha_s)^2 + \beta_d^2 + 4\beta_d\beta_s + \beta_s^2. \tag{A9}$$

Combined with formula (1), it is clear that this condition is naturally satisfied. Further, by substituting the number of two-sided users and their price in equilibrium into the profit functions, the equilibrium profits of the two platforms can be obtained:

$$\Pi_1' = \frac{\left(4 - 2\alpha_d\alpha_s - \beta_d^2 - 4\beta_d\beta_s - \beta_s^2\right)^2\left[4 - (\alpha_d + \alpha_s)^2 - \beta_d^2 - 6\beta_d\beta_s - \beta_s^2\right]}{4\left[2\left(3 - \beta_d^2 - 4\beta_d\beta_s - \beta_s^2\right) - \left(\alpha_d^2 + 4\alpha_d\alpha_s + \alpha_s^2\right)\right]^2};$$

$$\Pi_2' = \frac{\left[2 - (\alpha_d + \alpha_s)^2 - \beta_d^2 - 4\beta_d\beta_s - \beta_s^2\right]^2\left(4 - 4\alpha_d\alpha_s - \beta_d^2 - 6\beta_d\beta_s - \beta_s^2\right)}{4\left[2\left(3 - \beta_d^2 - 4\beta_d\beta_s - \beta_s^2\right) - \left(\alpha_d^2 + 4\alpha_d\alpha_s + \alpha_s^2\right)\right]^2}. \tag{A10}$$

From the above formula, it can be seen that the enhancement of network externalities motivates the platform to bring down the price and makes the number of platform users to increase, which eventually leads to a decline in the platform's profit. This is due to the intensified competition among platforms caused by the enhanced network externalities.

Since in the case of relatively high differentiation, $\overline{u}_i \in [0, 1]$, $\overline{x}_1 < 0$, $\overline{x}_2 > 1$ is satisfied, when combining the hypothesis of formula (1) and substituting the equilibrium results, the parameter conditions at this point satisfy:

$$\frac{(\alpha_d + \alpha_s)^2 + \beta_d^2 + 6\beta_d\beta_s + \beta_s^2}{8} \equiv \underset{-}{t} < t < t_2 \equiv \frac{1}{2}\left\{1 - \frac{2 + \alpha_d^2 + \alpha_s^2}{2\left(3 - \beta_d^2 - 4\beta_d\beta_s - \beta_s^2\right) - \left(\alpha_d^2 + 4\alpha_d\alpha_s + \alpha_s^2\right)}\right\}. \tag{A11}$$

From the above inequality condition, it can be drawn that the consumers on the two platforms may be distributed in a "trapezoidal" fashion, segmented into the upper and the lower parts, as shown in Figure 2.

Combining the above results, Lemma 2 can be obtained.

Lemma 2. In the case of strong differentiation ($\underset{-}{t} < t < t_2$), the following results can be obtained:

(1) When $\alpha_d^2 + \beta_d^2 > \alpha_s^2 + \beta_s^2, P' > 0$, i.e. the price of the bundles is non-negative; conversely, when the network externalities on the buyers' side are weaker than on the sellers' side, the price of the bundles is likely to be negative. And as the network externalities increase, the price of Platform 2 exhibits a fluctuation trend that goes positive-negative-positive.

(2) The price of the platform to content providers depends largely on the relative strength of the network externalities on both sides. When $\alpha_s > \alpha_d$, $q_1^{A'} > 0$; conversely, $q_1^{A'} \leq 0$; when $\beta_s > \beta_d$, $q_i^{B'} > 0$; conversely $q_i^{B'} \leq 0$.

(3) $\Pi_1' > \Pi_2' \geq 0$, i.e. the profits of both platforms are positive, and the profit of Platform 1 is always higher than that of Platform 2.

In contrast to Lemma 1, Lemma 2 depicts the equilibrium results of tying by Platform 1 when the transport cost is low. And similar to the case of weak differentiation, the price of Platform 1 for the bundles depends mainly on the relative strength of the network externalities between the buyers and the sellers in the two markets. But, unlike the previous scenario, when the transport cost is low, the competition between the two platforms is more intense, such that Platform 2's price to consumers has higher volatility and is influenced by the network externalities on both sides of Market A, even though the profits of the two platforms in equilibrium are positive. (*QED*)

Appendix 3

3.1. Comparison of equilibrium results in the case of strong differentiation

Comparing the relative changes in the two equilibrium results in the case of strong differentiation, we have the following relation:

$$\Delta P = \frac{\left(4t - \alpha_d\alpha_s - \alpha_s^2 - 3\beta_d\beta_s - \beta_s^2\right)(1 + M - 2\alpha_d\alpha_s)}{2\left[2M - \left(\alpha_d^2 + 4\alpha_d\alpha_s + \alpha_s^2\right)\right]} - \left[\frac{4t - \beta_s^2 - 3\beta_d\beta_s}{4} + \frac{2 - \alpha_s(\alpha_d + \alpha_s)}{4 - (\alpha_d + \alpha_s)^2}\right],$$

$$\Delta p_2 = \frac{\left[M - 1 - (\alpha_d + \alpha_s)^2\right]\left(4t - 2\alpha_d\alpha_s - 3\beta_d\beta_s - \beta_s^2\right)}{2\left[2M - \left(\alpha_d^2 + 4\alpha_d\alpha_s + \alpha_s^2\right)\right]} - \frac{4t - \beta_s^2 - 3\beta_d\beta_s}{4};$$

$$\Delta q_1^A = \frac{(\alpha_s - \alpha_d)\left[4(1 - \alpha_d\alpha_s) + (\alpha_d + \alpha_s)^2(1 + 2\alpha_d\alpha_s - M)\right]}{2\left[2M - \left(\alpha_d^2 + 4\alpha_d\alpha_s + \alpha_s^2\right)\right]};$$

$$\Delta q_1^B = -\Delta q_2^B = \frac{(\beta_s - \beta_d)\left(2 + \alpha_d^2 + \alpha_s^2\right)}{4\left[2M - \left(\alpha_d^2 + 4\alpha_d\alpha_s + \alpha_s^2\right)\right]}.$$

$$\Delta \Pi_1 = \frac{\left[M + 2t - (\alpha_d + \alpha_s)^2 - 2\beta_d\beta_s\right](M + 1 - 2\alpha_d\alpha_s)^2}{4\left[2M - \left(\alpha_d^2 + 4\alpha_d\alpha_s + \alpha_s^2\right)\right]^2} - \left[\frac{1}{4 - (\alpha_d + \alpha_s)^2} + \frac{\left(8t - \beta_d^2 - 6\beta_d\beta_s - \beta_s^2\right)}{16}\right],$$

$$\Delta \Pi_2 = \frac{(M + 2t - 4\alpha_d\alpha_s - 2\beta_d\beta_s)\left[M - 1 - (\alpha_d + \alpha_s)^2\right]^2}{4\left[2M - \left(\alpha_d^2 + 4\alpha_d\alpha_s + \alpha_s^2\right)\right]^2} - \frac{\left(8t - \beta_d^2 - 6\beta_d\beta_s - \beta_s^2\right)}{16}. \tag{A12}$$

Combining the above analysis, Lemma 3 can be obtained.

Lemma 3. In the case of strong differentiation, compared to the benchmark model, the results of tying by Platform 1 are as follows:

(1) When $\beta_d > 2\beta_s$ and $\beta_d \leq 2\beta_s$, and t is greater, $\Delta P < 0$; on the contrary, when $\beta_d \leq 2\beta_s$ and t is greater and α_j is weaker, $\Delta P > 0$, i.e. when the strength of the network externalities on the buyers' side is stronger than on the sellers' side, or the transport cost is relatively high, the price of the bundles will be lower than the sum of the prices of the two types of products corresponding to the benchmark model; on the contrary, when the network externalities on the sellers' side are stronger than on the buyers' side, the price of the bundles will be higher than the sum of the prices of the two types of products corresponding to the benchmark model. Similarly, when $\beta_d < \beta_s$ and t and α_j are sufficiently small, $\Delta p_2 > 0$; conversely, $\Delta p_2 < 0$.

(2) When $\alpha_s > \alpha_d$ and the transport cost is relatively high, $q_1^A > 0$; conversely, $q_1^A \leq 0$, i.e. when the network externalities on the sellers' side of the market are relatively stronger and the transport cost is higher, the price of content providers is lower than the corresponding price level in the benchmark model; in the opposite situation, it is higher than the corresponding price level in the benchmark model.

(3) When α_j and M are relatively small, $\Delta\Pi_1 > 0$, i.e. when the network externalities of Market A are weak and the transport costs are weaker than the network externalities on both sides of Market B, Platform 1 can increase its profit through tying; conversely, when α_j is large, $\Pi_1 < 0$, i.e. Platform 1 will have no incentive to tie when the network externalities of Market A are strong. In addition, $\Delta\Pi_2 < 0$, i.e. the profit of Platform 2 is always lower than that of Platform 1 when Platform 1 does not tie.

Proof. Combining the constraints of formulas (1) and (A5), the paper then calculates the changes in relative results under the two equilibria separately:

(1) the relative price change of bundles on Platform 1. First, calculate the partial derivative of ΔP with respect to α_j, we have $\partial\Delta P/(\partial\alpha_j) < 0$, i.e. the relative change in Platform 1's price to consumers is decreasing with respect to the network externalities on both sides in Market A. Therefore, from $\alpha_j = 0$, the paper examines the effects of the other parameters ΔP, when $sign\{\Delta P|_{(\alpha_j=0)}\} = sign\{\beta_d^2 + \beta_d\beta_s - 2t\}$. Combining formula (1) and (A5), the following results can be obtained: when $\beta_d > 2\beta_s$, or $\beta_d \leq 2\beta_s$ and t is relatively large, $\Delta P|_{(\alpha_j=0)} < 0$; on the contrary, when $\beta_d \leq 2\beta_s$ and t is relatively large, $\Delta P|_{(\alpha_j=0)} > 0$. Since ΔP is decreasing with respect to α_j, when $\beta_d > 2\beta_s$ or $\beta_d \leq 2\beta_s$ and t is relatively large, $\Delta P < 0$; while when $\beta_d \leq 2\beta_s$ and t is relatively large and α_j is relatively weak, $\Delta P > 0$.

(2) The relative change in Platform 2's price, p_2. Similarly, we calculate the partial derivative of Δp_2 with respect to α_j and have $\frac{\partial\Delta p_2}{\partial\alpha_j} < 0$; since $\Delta p2|_{(\alpha_j=0)} = -(4t - \beta_s^2 - 3\beta_d\beta_s)/4M$. Eventually, by combining formulas (1) and (A5), we have: when $\beta_d < \beta_s$ and t and α_j are sufficiently small, $\Delta p_2 > 0$; in the opposite situation, $\Delta p_2 < 0$.

(3) The relative change in the price paid by content providers in Market A, (q_1^A). From formula (A12), we have $sign\{\Delta q_1^A\} = sign\{(\alpha_s - \alpha_d)[4(1 - \alpha_d\alpha_s) + (\alpha_d + \alpha_s)^2(1 + 2\alpha_d\alpha_s - M)]\}$, i.e. when the network externalities on the sellers' side of the market are relatively stronger ($\alpha_s > \alpha_d$) and the transport cost is higher, thus satisfying $M > (4 + (\alpha_d - \alpha_s)^2)/(\alpha_d + \alpha_s)^2 + 2\alpha_d\alpha_s$, the content providers' price is lower than the corresponding price level in the benchmark model; conversely, it will be higher than the corresponding price level in the benchmark model. This suggests that the relative change in the price of Platform 1 to the content providers in Market A depends on the relative strength of the network externalities on both sides of Market A and the level of transport cost relative to the longer network externalities. In particular, when the network externalities on the content providers' side are relatively strong and the transport cost is sufficiently high, Platform 1 may increase its price to the content providers in the scenario of tying.

(4) The relative change in the price paid by content providers in Market B, (q_i^B). From formula (A12), it can be drawn that the relative change in the price paid by content providers in Market B depends solely on the strength of the network externalities on both sides of Market B, and the relative changes in the price paid by content providers on both platforms are of the same degree, but in opposite directions.

(5) The relative change in the profit of Platform 1. We calculate the partial derivative of Platform 1's profit, $\Delta\Pi_1$, with respect to α_j, and have $\partial\Delta\Pi_1 /\partial\alpha_j < 0$, i.e. the relative profit of platform 1 is decreasing with respect to α_j in both equilibria. Similar to the previous calculation, we measure the change in relative profit by comparing $\Delta\Pi_1|_{\alpha_j}$. Since $0 \leq \alpha_j \leq \sqrt{2}$, we substitute 0 and $\sqrt{2}$, and by comparing them respectively, it can be drawn that when α_j and M are both small, i.e. the network externalities of Market A are relatively weak and the transport cost is weaker than the network externalities on both sides of Market B, Platform 1 can increase its profit by tying; conversely, when the network externalities of Market A are relatively strong, Platform 1 will have no incentive to tie. Similarly, through comparison, the relative change in the profit of Platform 2 before and after Platform 1 chooses to tie can be obtained: by calculation, it can be seen that $\Delta\Pi_2$ is also decreasing with respect to α_j, and $\Delta\Pi_2|_{\alpha_j=0} < 0$, therefore, the profit of Platform 2 is always decreasing after tying. (*QED*)

Lemma 3 depicts the relative changes in prices and profits in the two equilibria. It can be seen that, compared with the benchmark model, the relative change in the price of bundles depends mainly on the relative strength of the network externalities on both sides of the platform in Market B; while the profit of Platform 1 depends on the strength of the network externalities on both sides of the platform in Market A and the value of M. Thus, under the analytical framework of this paper, when the platform is allowed to subsidize users on one side, Platform 1 can still increase its profit by price coordination, such that Platform 1 has an incentive to tie.

3. 2. Comparison of equilibrium results in the case of weak differentiation

Similar to the above analysis, the paper then compares the relative changes in the two equilibrium results in the case of weak differentiation, and we have the following relation:

$$\Delta P = \frac{\left(2-\alpha_d\alpha_s-\alpha_s^2-3\beta_d\beta_s-\beta_s^2\right)\left(4-2\alpha_d\alpha_s-\beta_d^2-4\beta_d\beta_s-\beta_s^2\right)}{2\left[2\left(3-\beta_d^2-4\beta_d\beta_s-\beta_s^2\right)-\left(\alpha_d^2+4\alpha_d\alpha_s+\alpha_s^2\right)\right]} - \left[\frac{4t-\beta_s^2-3\beta_d\beta_s}{4}+\frac{2-\alpha_s(\alpha_d+\alpha_s)}{4-(\alpha_d+\alpha_s)^2}\right],$$

$$\Delta p_2 = \frac{\left(2-2\alpha_d\alpha_s-3\beta_d\beta_s-\beta_s^2\right)\left[2-(\alpha_d+\alpha_s)^2-\beta_d^2-4\beta_d\beta_s-\beta_s^2\right]}{2\left[2\left(3-\beta_d^2-4\beta_d\beta_s-\beta_s^2\right)-\left(\alpha_d^2+4\alpha_d\alpha_s+\alpha_s^2\right)\right]} - \frac{4t-\beta_s^2-3\beta_d\beta_s}{4};$$

$$\Delta q_1^A = \frac{(\alpha_s-\alpha_d)}{4}\left\{\frac{2+\alpha_d^2+\alpha_s^2}{\left[2\left(3-\beta_d^2-4\beta_d\beta_s-\beta_s^2\right)-\left(\alpha_d^2+4\alpha_d\alpha_s+\alpha_s^2\right)\right]}-\frac{(\alpha_d+\alpha_s)^2}{4-(\alpha_d+\alpha_s)^2}\right\};$$

$$\Delta q_1^B = -\Delta q_2^B = \frac{\left(2+\alpha_d^2+\alpha_s^2\right)(\beta_s-\beta_d)}{4\left[2\left(3-\beta_d^2-4\beta_d\beta_s-\beta_s^2\right)-\left(\alpha_d^2+4\alpha_d\alpha_s+\alpha_s^2\right)\right]};$$

$$\Delta\Pi_1 = \frac{\left(4-2\alpha_d\alpha_s-\beta_d^2-4\beta_d\beta_s-\beta_s^2\right)^2\left[4-(\alpha_d+\alpha_s)^2-\beta_d^2-6\beta_d\beta_s-\beta_s^2\right]}{4\left[2\left(3-\beta_d^2-4\beta_d\beta_s-\beta_s^2\right)-\left(\alpha_d^2+4\alpha_d\alpha_s+\alpha_s^2\right)\right]^2} - \left[\frac{1}{4-(\alpha_d+\alpha_s)^2}+\frac{\left(8t-\beta_d^2-6\beta_d\beta_s-\beta_s^2\right)}{16}\right],$$

$$\Delta\Pi_2 = \frac{\left[2-(\alpha_d+\alpha_s)^2-\beta_d^2-4\beta_d\beta_s-\beta_s^2\right]^2\left(4-4\alpha_d\alpha_s-\beta_d^2-6\beta_d\beta_s-\beta_s^2\right)}{4\left[2\left(3-\beta_d^2-4\beta_d\beta_s-\beta_s^2\right)-\left(\alpha_d^2+4\alpha_d\alpha_s+\alpha_s^2\right)\right]^2} - \frac{\left(8t-\beta_d^2-6\beta_d\beta_s-\beta_s^2\right)}{16}. \quad \text{(A13)}$$

Combining the above analysis, Lemma 4 can be obtained.

Lemma 4. When the level of differentiation is relatively low ($t < t_2$), compared with the benchmark model, the equilibrium results of tying by Platform 1 are as follows:

(1) When α_j and β_j are both sufficiently small, $\Delta P > 0$, i.e. when the strength of the network externalities in both markets is weak, the price of the bundles will be higher than the sum of the corresponding prices in the benchmark model. In the opposite situation, $\Delta P \le 0$, i.e. when the network externalities of Market A or Market B are strong, the platform tends to set the assumption of the bundles at a level that is lower than the sum of the corresponding product prices in the benchmark model.

(2) When α_j is relatively small and $\beta_s > \beta_d$, $\Delta p_2 > 0$; conversely, $\Delta p_2 \leq 0$, suggesting that the relative change in Platform 2's price to consumers depends on the strength of the network externalities on both sides of Market A and the relative strength of the network externalities on both sides of Market B. When the strength of network externalities in Market A is relatively weak, and the network externalities on the sellers' side in Market B are stronger than those on the buyers' side, Platform 2 has an incentive to raise its price level; conversely, Platform 2 tends to set a price level lower than the benchmark model.

(3) When $\alpha_s > \alpha_d$, $\Delta q_1^A > 0$; conversely, $\Delta q_1^A \leq 0$, i.e. when the network externalities on the sellers' side are stronger than on the buyers' side, Platform 1's price to content providers in Market A will be higher than the corresponding price level in the benchmark model; on the contrary, when the network externalities on the buyers' side are relatively stronger, Platform 2 tends to set a price level lower than that in the benchmark model. Similarly, when $\beta_s > \beta_d$, $\Delta q_1^B > 0$ and $\Delta q_2^B < 0$; conversely, $\Delta q_1^B \leq 0$ and $\Delta q_2^B \geq 0$, i.e. the two platforms' price to content providers in Market B is determined by the relative strength of the network externalities on both sides, and their price fluctuates to the same extent and in opposite directions.

(4) When α_j is relatively small and $(\beta_d + \beta_s) \to \sqrt{6}/2$ and $\beta_d\beta_s < 1$, $\Delta\Pi_1 > 0$, i.e. when the strength of the network externalities is weak in both markets, and the network externalities on both sides in Market B have relatively strong asymmetry, Platform 1 is able to increase its profit level by tying; in the opposite situation, Platform 1 will have no incentive to tie, and the profit of Platform 2 is always decreasing.

Proof. According to the constraints of formulas (A9) and (A11), we have $0 \leq \alpha_j \leq \sqrt{2}$, $0 \leq \beta_j \leq \sqrt{2}$, $t < 1/2, j = \{d, s\}$. Therefore, by combining the constraints of formula (1), the paper will then calculate and compare the relative changes in the equilibrium price of the platforms and their profits under each of the two equilibria:

(1) by calculating the partial derivative of ΔP with respect to α_j, and combining the constraints of formulas (1) and (32), we have $\partial \Delta P/\partial \alpha_j < 0$ and $\Delta P|_{\alpha_j=\sqrt{2}} < 0$, i.e. the relative change in the price of the bundles is decreasing with respect to the network externalities on both sides of Market A, and it is negative when the network externalities reach the strongest in Market A, i.e. subsidizing the users. From this, we can confirm the relative price change of the bundles by examining the value of ΔP when $\alpha_j = 0$: $sign\{\Delta P|_{\alpha_j=0}\} = sign\{(2 - \beta_d\beta_s - \beta_s^2) - 4t(3 - \beta_d^2 - 4\beta_d\beta_s - \beta_s^2)\}$. It is clear that when β_j is relatively high and let $(2 - \beta_d\beta_s - \beta_s^2) \to 0$, $\Delta P|_{\alpha_j=0} < 0$. Since ΔP is decreasing with respect to α_j, thus $\Delta P < 0$, i.e. at this point, the price of the bundles is lower than the sum of the prices of the corresponding two products when sold separately; conversely, when both β_j and t are sufficiently small, $\Delta P|_{\alpha_j=0} > 0$, at this point, there will be $\overline{\alpha}_j$ that satisfies $\Delta P(\alpha_j = \overline{\alpha}_j)$ when $\Delta P = 0$, therefore, when $\alpha_j < \overline{\alpha}_j$ and β_j is sufficiently small, $\Delta P > 0$, and in the opposite situation, $\Delta P \leq 0$.

(2) Similarly, through comparison, it can be drawn that Δp_2 is decreasing with respect to α_j and is negative when α_j takes its maximum value, which can be measured by examining the positive and negative results of $\Delta p_2|_{\alpha_j=0}$, $sign\{\Delta p_2|_{\alpha_j=0}\} = sign\{(3 - \beta_d^2 - 4\beta_d\beta_s - \beta_s^2)(2 - 4t) - (2 - \beta_d\beta_s - \beta_s^2)\}$. We know from formula (A11) that $t < 1/2$, the first term on the right side of the equation is positive, thus $(3 - \beta_d^2 - 4\beta_d\beta_s - \beta_s^2)(2 - 4t) - (2 - \beta_d\beta_s - \beta_s^2) > 1 - \beta_d\beta_s - \beta_s^2$. Combining with formula (1), it can be drawn that when $\beta_s > \beta_d$, $\Delta p_2|_{\alpha_j=0} > 0$; in the opposite situation, $\Delta p_2|_{\alpha_j=0}$ may be less than 0. Therefore, when $\alpha_j \to 0$ and $\beta_s > \beta_d$, $\Delta p_2 > 0$; conversely, $\Delta p_2 \leq 0$.

(3) Since the second term of Δq_A^1 is increasing with respect to α_j and is positive when $\alpha_j = 0$, we have $sign\{\Delta q_A^1\} = sign\{\alpha_s - \alpha_d\}$, i.e. the relative change of Platform 1's price to content providers in Market A mainly depends on the relative strength of the network externalities on both sides of the platform: when the network externalities on the sellers' side are stronger than on the buyers' side, i.e. when $\alpha_s > \alpha_d$, Platform 1 will raise its price to content providers after

bundling, as compared with the benchmark model; conversely, when the network externalities on the buyers' side are stronger than on the sellers' side, i.e. when $\alpha_s < \alpha_d$, Platform 1 will lower its price to content providers.

(4) Similar to Market A, the relative change of the two platforms' prices to content providers in Market B is mainly determined by $(\beta_s - \beta_d)$. When $\beta_s > \beta_d$, Platform 1 will raise its price to content providers after tying, while Platform 2 will bring down its price to content providers to the same extent; conversely, when $\beta_s < \beta_d$, Platform 1 tends to reduce its charge to content providers as compared to the benchmark model, while Platform 2 tends to increase its charge to content providers.

(5) By calculation and comparison, it is shown that $\Delta\Pi_1$ is decreasing with respect to α_j, and is negative when α_j takes its maximum value, thus, the incentive for platform 1 to tie can be determined by examining the value of $\Delta\Pi_1$ when $\alpha_j = 0$. Combining $t < 1/2$, we obtain $sign\{\Delta\Pi_1|_{\alpha_j=0}\} = sign\{1 - \beta_d\beta_s - (3 - \beta_d^2 - 4\beta_d\beta_s - \beta_s^2)[3 - 2(\beta_d + \beta_s)^2]\}$. The comparison shows that when $(\beta_d + \beta_s) \to \sqrt{6}/2$ and $\beta_d\beta_s < 1$, the second term on the right side of the equation approaches 0, thus $\Delta\Pi_1|_{\alpha_j=0} > 0$. Therefore, when α_j is relatively small and $(\beta_d + \beta_s) \to \sqrt{6}/2$, Platform 1 will have an incentive to tie; conversely, Platform 1 will have no incentive to tie. Similarly, the relative change in Platform 2's profit $\Delta\Pi_2$ is also decreasing with respect to α_j and is negative when α_j takes its maximum value, and after further calculation, we have $sign\{\Delta\Pi_2|_{\alpha_j=0}\} = sign\{-(5 - 2\beta_d^2 - 8\beta_d\beta_s - 2\beta_s^2)\} < 0$, i.e. the platform's profit is always decreasing. *(QED)*

Lemma 4 shows that the strength of the network externalities and their degree of symmetry have a vital impact on the relative change in the equilibrium results. In particular, for the relative change in the price of the bundles relative to the sum of the prices of the two corresponding products in the benchmark model, when one of the two markets (e.g. Market B) has relatively strong network externalities on both sides, Platform 1 has an incentive to set the price lower than the sum of the corresponding prices in the benchmark model in order to attract consumers to buy the bundles. At this point, if the network externalities on both sides of Market B have a strong asymmetry, Platform 1 will have an incentive to tie. At this point, compared with the benchmark model, as the price of the bundles decreases, its price to the content providers instead increases, thus elevating its overall profit. *(QED)*

Appendix 4

The study in this paper is based on Choi and Jeon (2021), and by relaxing the non-negative price constraint, this paper explores the incentives of two-sided platforms to leverage market power by tying in a situation where the bundled item market has horizontal differences. In Choi and Jeon (2021), the mechanism by which the leverage of tying works is that tying enables two-sided platforms to gain additional revenue and the presence of the non-negative price constraint prevents this revenue from being consumed by competition. Iacobucci and Ducci (2019), on the other hand, provide a different perspective on the leverage of tying of two-sided platforms in the form of numerical examples and highlight that tying helps the monopolistic platform to leverage its market power in the primary product to the bundled product market and gain additional revenue by attracting users from the other side (e.g. advertisers).

In order to clarify the mechanism by which the above study of leverage of tying works and its relevance to this paper, the simple model proposed by Choi and Jeon (2021) is introduced below.

Suppose there exist two Markets A and B and two two-sided platforms (Platform 1 and Platform 2). Market A is one-sided, and consumers' evaluation of Product A in Market A is equal, written as u. Platform 1 is in a monopoly position in Market A; both platforms provide transaction services for consumers and content providers, causing Competitive Bottlenecks in Market B, i.e. homogeneous consumers with unit demand only buy products in one platform, while homogeneous content providers have access to both platforms; the total number of both consumers and content providers is 1, and the marginal revenue gained by content providers from each consumer is β, i.e. the strength of the network externalities. Meanwhile, both platforms offer the product with quality differences (B_i). Consumers' evaluation of product B_i is $v_i, i \in \{1, 2\}$, and $v_2 - v_1 = \Delta > 0, u > \Delta\ ,\ \beta > \Delta$. Following this, the paper compares the incentives for Platform 1 to tie in the presence and absence of the non-negative price constraint. (1) In the presence of non-negative price constraint, and when Platform 1 does not tie, Platform 1 charges u for Product A and gains the profit of u; since Platform 2 is of higher quality, Platform 1's charge in Market B is 0 and thus making a profit of 0, while Platform 2 charges Δ, and the profit is $(\Delta + \beta)$. When Platform 1 chooses to tie, the competition between the two platforms will make the charge for Platform 2 to 0, and Platform 1's price for the bundles is $(u - \Delta)$, therefore, the profit of Platform 1 is $(u + \beta - \Delta)$, and the profit of Platform 2 is 0. A comparison of the profit changes of Platform 1 before and after tying shows that Platform 1's profit increases after tying, and Platform 1's "price" for Product B at this point is $\beta - \Delta > 0$, thus Platform 1 effectively leverages market power by tying the two products. (2) In the absence of non-negative price constraint, the profit of Platform 1 remains unchanged at u when there is no tying; when Platform 1 chooses to tie, Platform 2 will set the price at $-\beta$ and obtain profit 0; Platform 1's price for the bundles is $(u - \Delta - \beta)$ and obtains profit $(u - \Delta) < u$. It is clear from the comparison that Platform 1 will have no incentive to tie.

In the above model, the two sides of the platform in Market B are composed of "*homogeneous consumers and homogeneous content providers*." In order to explore the conditions under which Platform 1 has an incentive to tie when the price is negative, this paper expands and compares the three aspects of "*horizontally differentiated consumers and homogeneous content providers*," "*homogeneous consumers and heterogeneous content providers*" and "*horizontally differentiated consumers and heterogeneous content providers*," respectively. In particular, the "*horizontal differentiation of consumers*" is depicted by the Hotelling model, while the "*heterogeneous content providers*" follow the setting in this paper, i.e. the assumption that content providers face heterogeneous installation costs. The calculation results show that Platform 1 has an incentive to tie only when consumers are horizontally differentiated and content providers are heterogeneous in market B, and the equilibrium condition is not affected by the non-negative price constraint. The logic behind this is that the presence of horizontal differentiation moderates the competition among platforms and allows more content providers to access Platform 1, enabling Platform 1 to compensate for its loss on the consumers' side by raising its price to content providers; at the same time, heterogeneity on the sellers' side allows Platform 1 to transfer its quantitative advantage on the buyers' side to the content providers' side and, therefore, gains a competitive advantage. *(QED)*

Corresponding author
Cui Wei can be contacted at: wei.cui16@gmail.com