Confidence intervals for functions of signal-to-noise ratio with application to economics and finance

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Abstract

Purpose – Confidence intervals play a crucial role in economics and finance, providing a credible range of values for an unknown parameter along with a corresponding level of certainty. Their applications encompass economic forecasting, market research, financial forecasting, econometric analysis, policy analysis, financial reporting, investment decision-making, credit risk assessment and consumer confidence surveys. Signal-to-noise ratio (SNR) finds applications in economics and finance across various domains such as economic forecasting, financial modeling, market analysis and risk assessment. A high SNR indicates a robust and dependable signal, simplifying the process of making well-informed decisions. On the other hand, a low SNR indicates a weak signal that could be obscured by noise, so decision-making procedures need to take this into serious consideration. This research focuses on the development of confidence intervals for functions derived from the SNR and explores their application in the fields of economics and finance.

Design/methodology/approach – The construction of the confidence intervals involved the application of various methodologies. For the SNR, confidence intervals were formed using the generalized confidence interval (GCI), large sample and Bayesian approaches. The difference between SNRs was estimated through the GCI, large sample, method of variance estimates recovery (MOVER), parametric bootstrap and Bayesian approaches. Additionally, confidence intervals for the common SNR were constructed using the GCI, adjusted MOVER, computational and Bayesian approaches. The performance of these confidence intervals was assessed using coverage probability and average length, evaluated through Monte Carlo simulation.

Findings – The GCI approach demonstrated superior performance over other approaches in terms of both coverage probability and average length for the SNR and the difference between SNRs. Hence, employing the GCI approach is advised for constructing confidence intervals for these parameters. As for the common SNR, the Bayesian approach exhibited the shortest average length. Consequently, the Bayesian approach is recommended for constructing confidence intervals for the common SNR.

Originality/value – This research presents confidence intervals for functions of the SNR to assess SNR estimation in the fields of economics and finance.

Keywords Average length, Confidence interval, Coverage probability, Monte Carlo simulation, Signal-to-noise ratio

Paper type Research paper

1. Introduction

Confidence intervals play a crucial role in economics and finance, providing a reliable range of values for an unknown parameter along with a specified level of certainty. Here are diverse

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applications of confidence intervals in these fields. Economic forecasting: confidence intervals are essential for projecting economic metrics like gross domestic product (GDP) growth, inflation rates, and unemployment rates, providing a spectrum of values for the likely actual figures. Market research: in finance, confidence intervals help gauge the potential range of returns on investments. Analysts use them to convey confidence regarding future stock prices or returns on financial instruments. Risk management: confidence intervals play a pivotal role in evaluating and managing financial risk. They assist in approximating potential losses in investment portfolios, enabling informed decisions by investors and financial institutions. Financial forecasting: Confidence intervals are incorporated in financial modeling to project future cash flows, interest rates, and other financial parameters, enhancing the accuracy of predictions about the prospective financial performance of companies. Econometric analysis: in econometrics, confidence intervals gauge the precision of regression coefficients and other model parameters, which is crucial for determining the statistical significance of relationships between economic variables. Policy analysis: Economists use confidence intervals when scrutinizing the repercussions of policy changes, estimating the impact of a tax policy on consumer spending, and providing a confidence interval to convey associated uncertainty. Financial reporting: confidence intervals find application in financial statement analysis to estimate the precision of financial ratios, contributing to the assessment of the financial health and performance of companies. Investment decision-making: Investors rely on confidence intervals to assess potential returns and risks linked to diverse investment opportunities, aiding in making well-informed decisions concerning asset allocation and portfolio management. Credit risk assessment: in banking and finance, confidence intervals are used to evaluate credit risk, estimating the potential range of default probabilities, and establishing suitable interest rates for loans. Consumer confidence surveys: Confidence intervals are employed in the analysis and interpretation of survey data, such as consumer confidence surveys, providing a measure of uncertainty around reported confidence levels. In conclusion, confidence intervals serve as a valuable tool in economics and finance, offering a method to quantify and convey uncertainty in various analyses and decision-making processes.

The signal-to-noise ratio (SNR) in the realms of economics and finance originates from signal processing, representing the proportion of valuable information, termed the signal to irrelevant or random background noise. In the context of economic and financial analysis, this concept is commonly utilized to evaluate the information’s quality and the signal’s strength compared to the surrounding noise. Within economic and financial analysis, the term signal denotes meaningful and pertinent data or patterns, while noise pertains to random fluctuations or inconsequential information. The SNR functions as a metric for assessing the clarity and dependability of a signal amidst background noise. The applications of SNR in economics and finance extend across various domains, encompassing economic forecasting, financial modeling, market analysis, and risk assessment. A high SNR suggests a robust and dependable signal, facilitating more straightforward decision-making. Conversely, a low SNR implies a weak signal, potentially obscured by noise, necessitating careful consideration in decision-making processes. In essence, comprehending and managing the SNR is paramount for extracting meaningful insights and making informed decisions within economic and financial contexts.

Point estimation involves providing a single, specific value as an estimate for an unknown parameter in a population. For example, estimate the population mean based on a sample mean. Interval estimation, on the other hand, provides a range of values (an interval) within which the true parameter is likely to lie. This is typically expressed as a confidence interval. Interval estimation is often considered better than point estimation. This is because it incorporates uncertainty, confidence level, decision-making, and robustness. To incorporate uncertainty, interval estimation explicitly acknowledges the uncertainty inherent in estimating population parameters from a sample. It provides a sense of the range of plausible values. For confidence level, confidence intervals come with a specified confidence level (e.g., 95%). This indicates the proportion of intervals from repeated sampling that would include the true parameter. It offers a clear indication of the reliability of the estimate. For decision-making purposes, having a range of values is often more informative than a single point. It allows decision-makers to consider a spectrum of possibilities. For robustness, point estimates can be sensitive to outliers or extreme values in the data. Confidence intervals, especially those based on robust methods, may be less affected by extreme observations.

A confidence interval for a parameter of interest is a statistical range that provides an estimated range of values that is likely to include the true value of the parameter. It is constructed based on the sample data and is associated with a certain level of confidence. The confidence interval is a measure of the precision or uncertainty of the estimated parameter. For example, if you are estimating the SNR of a population, a 95% confidence interval would imply that if you were to take many samples and construct a confidence interval from each, about 95% of those intervals would contain the true population SNR. Moreover, the confidence interval for the difference between parameters of interest is a range of values that is likely to contain the true difference between two population parameters. This type of interval estimation is commonly used in statistical analysis, especially when comparing two groups or assessing the impact of an intervention. In addition, the confidence interval for a common parameter of interest is an interval estimate that provides a range of plausible values for the true value of a parameter. This type of interval estimation is commonly used in statistical analysis when dealing with a single population parameter.

2. Literature review

The Generalized Confidence Interval (GCI) approach is designed to be versatile across diverse data types and statistical scenarios. It is not constrained by specific distributional assumptions, making it applicable in situations where classical methods may not be appropriate. The GCI has diverse applications in fields such as economics, finance, biology, and any domain requiring statistical inference. This methodology utilizes the generalized pivotal quantity (GPQ) to construct the confidence interval, enabling the estimation of confidence intervals for complex parameters. However, it's important to note that the numerical simulation of the GCI approach relies solely on the maximum likelihood estimate. Many researchers have undertaken comparisons between the GCI approach and alternative methods for constructing confidence intervals, as evidenced in studies by Weerahandi (1993), Krishnamoorthy and Lu (2003), Krishnamoorthy and Mathew (2003), Tian (2005), Chen and Zhou (2006), Tian and Wu (2007), Ye *et al.* (2010), Saothayanun and Thangjai (2018), Thangjai and Niwitpong (2019), Thangjai and Niwitpong (2020a), and Thangjai and Niwitpong (2020b).

Constructing confidence intervals using the large sample approach involves exploiting asymptotic properties, particularly when dealing with a substantial volume of data. This method relies on the Central Limit Theorem, which posits that the distribution of sample means converges to a normal distribution as the sample size increases. Utilizing this principle allows for the estimation of confidence intervals under the assumption of normality, enhancing their applicability in extensive datasets. The large sample approach is advantageous due to its simplicity in constructing the confidence interval using the exact formula. However, a limitation is that it requires a large sample size for estimating the confidence interval. Several scholars have evaluated the large sample approach in comparison to alternative methods for constructing confidence intervals, as demonstrated in the research conducted by Tian and Wu (2007), Saothayanun and Thangjai (2018), Thangjai and Niwitpong (2019), and Thangjai and Niwitpong (2020b).

The MOVER approach relies on the original confidence interval for a specific parameter of interest to derive the final confidence interval. An advantage of the MOVER approach is its ease of computation using the exact formula. However, a drawback is that it can be constructed with or without the initial confidence interval for a single parameter of interest. Several researchers, including Zou and Donner (2008), Zou *et al.* (2009), Saothayanun and Thangjai (2018), Thangjai and Niwitpong (2019), and Thangjai and Niwitpong (2020b), have recommended the utilization of the MOVER approach in constructing confidence intervals.

The adjusted MOVER approach is inspired by the principles of both the large sample and MOVER approaches. Its advantage lies in the straightforward application of the exact formula for confidence interval computation, although a drawback is that it relies on the initial confidence interval for a single parameter. Thangjai and Niwitpong (2020a) has delved into the investigation of the adjusted MOVER approach.

The bootstrap approach involves approximating the sampling distribution of statistics by iteratively resampling with replacements from the population. These multiple bootstrap samples, drawn from the population on numerous occasions, function as representative samples of the entire population. The bootstrap approach provides a simple and reasonably accurate technique for constructing confidence intervals. However, a drawback is the requirement for knowledge regarding the distribution of estimates around the true values because the sampling distribution aligns with the data distribution, considering that the estimates are derived from the data. Various researchers, including Chachi (2017) and Thangjai and Niwitpong (2020b), have advocated for the utilization of the bootstrap approach.

The computational approach is employed to formulate confidence intervals for intricate parameters. This technique involves simulations and numerical computations utilizing the maximum likelihood estimate. Scholars have introduced the computational approach for confidence intervals, as demonstrated in works by Pal *et al.* (2007) and Thangjai and Niwitpong (2020a).

The Bayesian approach employs posterior probability and facilitates comparison with alternative methods for constructing credible intervals. The primary motivation for opting for the Bayesian approach is the complexity of models that traditional methods may struggle to address. It is essential to emphasize that, irrespective of the rationale behind adopting the Bayesian approach, conducting a sensitivity analysis of priors is always crucial and should be included. This comparison is substantiated by studies such as Rao and D'Cunha (2016) and Ma and Chen (2018).

3. Methodology

The SNR can be described as the reciprocal of the coefficient of variation. The SNR is calculated as the ratio of the mean to the standard deviation. This paper discussed three parts as follow: The SNR, the difference between SNRs, and the common SNR.

3.1 Confidence intervals for the SNR

Suppose that random sample $X = (X_1, X_2, \ldots, X_n)$ follows any distribution. Suppose that μ and σ are population mean and population standard deviation of the distribution, respectively. The SNR is defined as

$$\theta = \frac{\mu}{\sigma}. \tag{1}$$

Let $\overline{X}$ and S are sample mean and sample standard deviation of the distribution, respectively. The estimator of the SNR is defined as

$$\hat{\theta} = \frac{\overline{X}}{S}. \tag{2}$$

3.1.1 GCI approach for SNR. The concept of GCI was introduced by Weerahandi (1993). Let $X = (X_1, X_2, \ldots, X_n)$ be a random sample having a density function $f(X|\theta, \upsilon)$, where θ is the parameter of interest and υ is a nuisance parameter. Let x be the observed sample of X. A generalized pivotal quantity $R(X;\ x, \theta, \upsilon)$ is considered and satisfies the following conditions:

(i) The distribution of $R(X;\ x, \theta, \upsilon)$ is free of all unknown parameters.

(ii) The observed value of $R(X;\ x, \theta, \upsilon)$ is the parameter of interest.

Condition (i) is imposed to guarantee that a subset of the sample space of the possible values of $R(X;\ x, \theta, \upsilon)$ can be found at a given value of the confidence coefficient with no knowledge of the parameters. Condition (ii) is imposed to ensure that such probability statements based on the GPQ lead to confidence regions involving observed data x only. The GCI for θ is computed using the percentiles of the GPQ. Let $[R(\alpha/2), R(1-\alpha/2)]$ be a $100(1-\alpha)\%$ two-sided GCI for the parameter of interest, where $R(\alpha/2)$ and $R(1-\alpha/2)$ denote the $100(\alpha/2)$-th and the $100(1-\alpha/2)$-th percentiles of $R(X;\ x, \theta, \upsilon)$, respectively.

Following Saothayanun and Thangjai (2018). Let R_μ be the GPQ of μ and let R_σ be the GPQ of σ. The GPQ of θ is defined as

$$R_\theta = \frac{R_\mu}{R_\sigma}. \tag{3}$$

The $100(1-\alpha)\%$ two-sided confidence interval for the SNR based on the GCI approach is given by

$$CI_{\theta.GCI} = [L_{\theta.GCI}, U_{\theta.GCI}] = [R_\theta(\alpha/2), R_\theta(1-\alpha/2)], \tag{4}$$

where $R_\theta(\alpha/2)$ and $R_\theta(1-\alpha/2)$ denote the $(\alpha/2)$-th and $(1-\alpha/2)$-th quantiles of R_θ, respectively.

The following algorithm is used to construct the GCI for the SNR.

Algorithm 1. For a given $\bar{x}$ and s
For $g = 1$ to m
Compute R_μ
Compute R_σ
Compute R_θ
End g loop
Compute the $(\alpha/2)$-th quantiles of R_θ defined by $R_\theta(\alpha/2)$
Compute the $(1-\alpha/2)$-th quantiles of R_θ defined by $R_\theta(1-\alpha/2)$

3.1.2 Large sample approach for SNR. According to Saothayanun and Thangjai (2018) the $100(1-\alpha)\%$ two-sided confidence interval for the SNR based on the large sample approach is given by

$$CI_{\theta.LS} = [L_{\theta.LS}, U_{\theta.LS}] = \left[\widehat{\theta} - z_{1-\alpha/2}\sqrt{\mathrm{Var}\left(\widehat{\theta}\right)}, \widehat{\theta} + z_{1-\alpha/2}\sqrt{\mathrm{Var}\left(\widehat{\theta}\right)}\right], \tag{5}$$

where $z_{1-\alpha/2}$ denotes the $(1-\alpha/2)$-th quantile of a standard normal distribution and $\mathrm{Var}(\widehat{\theta})$ is the variance of the estimator of SNR.

3.1.3 Bayesian approach for SNR. Bayes' rule is utilized to revise the prior distribution, resulting in the posterior distribution, which encompasses all relevant information regarding the unknown parameters inferred from the observed data. The Bayesian approach provides a framework for adjusting beliefs and making predictions based on new evidence or data. It is grounded in Bayes' theorem, which integrates prior probability and likelihood to compute the posterior probability. The prior distribution reflects uncertainty about parameters before observing the data. In this study, we utilized Jeffreys' independence prior.

Let $\sigma|x$ be the posterior distribution of σ. And let $\mu|\sigma, x$ be the posterior distribution of μ given σ. Let θ_{BS} be the posterior distribution using $\sigma|x$ and $\mu|\sigma, x$.

The $100(1-\alpha)\%$ two-sided confidence interval for the SNR based on the Bayesian approach is given by

$$CI_{\theta.BS} = [L_{\theta.BS}, U_{\theta.BS}], \tag{6}$$

where $L_{\theta.BS}$ and $U_{\theta.BS}$ are the lower and upper limits of the shortest $100(1-\alpha)\%$ highest posterior density interval of θ_{BS}, respectively.

The following algorithm is used to construct the Bayesian credible interval for the SNR.

Algorithm 2. For a given $\overline{x}$ and s
For $g = 1$ to m
Compute $\sigma|x$
Compute $\mu|\sigma, x$
Compute θ_{BS}
End g loop
Compute the shortest $100(1-\alpha)\%$ highest posterior density interval of θ_{BS}

The following algorithm is used to evaluate the coverage probabilities and average lengths of the confidence intervals for SNR.

Algorithm 3. For a given μ, σ, and θ
For $h = 1$ to M
Generate x
Calculate $\overline{x}$ and s
Construct the confidence interval $[L_{\theta.GCI}, U_{\theta.GCI}]$
Construct the confidence interval $[L_{\theta.LS}, U_{\theta.LS}]$
Construct the confidence interval $[L_{\theta.BS}, U_{\theta.BS}]$
If $L \le \theta \le U$, set $p = 1$; else set $p = 0$
Compute $U - L$
End h loop
Compute mean of p defined by the coverage probability
Compute mean of $U - L$ defined by the average length

3.2 Confidence intervals for the difference between SNRs

Suppose that $X = (X_1, X_2, ..., X_n)$ follows any distribution with mean μ_X and standard deviation σ_X. Similarly, let $Y = (Y_1, Y_2, ..., Y_m)$ be any distribution with mean μ_Y and standard deviation σ_Y. Moreover, X and Y are independent. The single SNRs of X and Y are given by

$$\theta_X = \frac{\mu_X}{\sigma_X} \text{ and } \theta_Y = \frac{\mu_Y}{\sigma_Y}. \tag{7}$$

The difference between of SNRs is defined as

$$\delta = \theta_X - \theta_Y. \tag{8}$$

Let $\overline{X}$ and S_X are sample mean and sample standard deviation of X, respectively. Moreover, let $\overline{Y}$ and S_Y are sample mean and sample standard deviation of Y, respectively. Suppose that $\widehat{\theta}_X$ and $\widehat{\theta}_Y$ are the estimators of θ_X and θ_Y, respectively, which are given

$$\widehat{\theta}_X = \frac{\overline{X}}{S_X} \text{ and } \widehat{\theta}_Y = \frac{\overline{Y}}{S_Y}. \tag{9}$$

The difference between of SNRs is defined as

$$\widehat{\delta} = \widehat{\theta}_X - \widehat{\theta}_Y. \tag{10}$$

Suppose that $\mathrm{Var}(\widehat{\theta}_X)$ and $\mathrm{Var}(\widehat{\theta}_Y)$ are the variances of $\widehat{\theta}_X$ and $\widehat{\theta}_Y$, respectively. The variance of $\widehat{\delta} = \widehat{\theta}_X - \widehat{\theta}_Y$ is

$$\mathrm{Var}\left(\widehat{\delta}\right) = \mathrm{Var}\left(\widehat{\theta}_X - \widehat{\theta}_Y\right) = \mathrm{Var}\left(\widehat{\theta}_X\right) + \mathrm{Var}\left(\widehat{\theta}_Y\right). \tag{11}$$

3.2.1 GCI approach for the difference between SNRs. According to Thangjai and Niwitpong (2019) and Thangjai and Niwitpong (2020b). Let R_{μ_X} be the GPQ of μ_X and let R_{σ_X} be the GPQ of σ_X. The GPQ of θ_X is defined as

$$R_{\theta_X} = \frac{R_{\mu_X}}{R_{\sigma_X}}. \tag{12}$$

Moreover, let R_{μ_Y} be the GPQ of μ_Y and let R_{σ_Y} be the GPQ of σ_Y. The GPQ of θ_Y is defined as

$$R_{\theta_Y} = \frac{R_{\mu_Y}}{R_{\sigma_Y}}. \tag{13}$$

Therefore, the difference between the GPQs of SNRs is

$$R_\delta = R_{\theta_X} - R_{\theta_Y}. \tag{14}$$

The $100(1-\alpha)\%$ two-sided confidence interval for the difference between SNRs based on the GCI approach is given by

$$CI_{\delta.GCI} = [L_{\delta.GCI}, U_{\delta.GCI}] = [R_\delta(\alpha/2), R_\delta(1-\alpha/2)], \tag{15}$$

where $R_\delta(\alpha/2)$ and $R_\delta(1-\alpha/2)$ denote the $(\alpha/2)$-th and $(1-\alpha/2)$-th quantiles of R_δ, respectively.

The following algorithm is used to construct the GCI for the difference between SNRs.

Algorithm 4. For a given $\bar{x}$, $\bar{y}$, s_X and s_Y
For $g = 1$ to m
Compute R_{μ_X}, R_{σ_X}, and R_{θ_X}
Compute R_{μ_Y}, R_{σ_Y}, and R_{θ_Y}
Compute R_δ
End g loop
Compute the $(\alpha/2)$-th quantiles of R_δ defined by $R_\delta(\alpha/2)$
Compute the $(1-\alpha/2)$-th quantiles of R_δ defined by $R_\delta(1-\alpha/2)$

3.2.2 Large sample approach for the difference between SNRs. Following Thangjai and Niwitpong (2019) and Thangjai and Niwitpong (2020b) using the central limit theorem, the $100(1-\alpha)\%$ two-sided confidence interval for the difference between SNRs based on the large sample approach is given by

$$CI_{\delta.LS} = [L_{\delta.LS}, U_{\delta.LS}] = \left[\widehat{\delta} - z_{1-\alpha/2}\sqrt{\mathrm{Var}\left(\widehat{\delta}\right)}, \widehat{\delta} + z_{1-\alpha/2}\sqrt{\mathrm{Var}\left(\widehat{\delta}\right)}\right], \tag{16}$$

where $z_{1-\alpha/2}$ is the $(1-\alpha/2)$-th quantile of the standard normal distribution and $\mathrm{Var}(\widehat{\delta})$ is the variance of the estimator of difference between SNRs.

3.2.3 MOVER for the difference between SNRs. Let l_X and u_X be the lower and upper limits of the confidence interval for SNR of X, respectively. Similarly, let l_Y and u_Y be the lower and upper limits of the confidence interval for SNR of Y, respectively.

Following Zou and Donner (2008), Zou *et al.* (2009), Thangjai and Niwitpong (2019) and Thangjai and Niwitpong (2020b), the $100(1-\alpha)\%$ two-sided confidence interval for the difference between the SNRs based on the MOVER approach is given by

$$\mathrm{CI}_{\delta.\mathrm{MOVER}} = [\mathrm{L}_{\delta.\mathrm{MOVER}}, \mathrm{U}_{\delta.\mathrm{MOVER}}]$$

$$= \left[\widehat{\theta}_X - \widehat{\theta}_Y - \sqrt{\left(\widehat{\theta}_X - l_X\right)^2 + \left(u_Y - \widehat{\theta}_Y\right)^2}, \widehat{\theta}_X - \widehat{\theta}_Y + \sqrt{\left(u_X - \widehat{\theta}_X\right)^2 + \left(\widehat{\theta}_Y - l_Y\right)^2}\right]. \tag{17}$$

3.2.4 Parametric bootstrap approach for the difference between SNRs. The parametric bootstrap approach is a resampling approach based on independently sampling with a replacement from existing sample data of the same sample size.

Let $X^* = (X_1^*, X_2^*, \ldots, X_n^*)$ be sample with replacement from $X = (X_1, X_2, \ldots, X_n)$ with sample size n and let $x^* = (x_1^*, x_2^*, \ldots, x_n^*)$ be the observed values of $X^* = (X_1^*, X_2^*, \ldots, X_n^*)$. Similarly, let $Y^* = (Y_1^*, Y_2^*, \ldots, Y_m^*)$ be the sample from $Y = (Y_1, Y_2, \ldots, Y_m)$ with replacement sample size m and let $y^* = (y_1^*, y_2^*, \ldots, y_m^*)$ be the observed values of $Y^* = (Y_1^*, Y_2^*, \ldots, Y_m^*)$.

The re-sampled sample is called a bootstrap sample. The difference in SNRs from the bootstrap sample is obtained by

$$\delta^* = \theta_X^* - \theta_Y^*. \tag{18}$$

An estimator of the difference of SNRs is

$$\widehat{\delta}^* = \widehat{\theta}_X^* - \widehat{\theta}_Y^*. \tag{19}$$

For replicate B times, there are totally B estimates of the difference of SNRs δ^* from B bootstrap sample.

The sampling distribution is constructed with these B bootstrap statistics. The confidence interval for the difference in SNRs is calculated using the distribution. Therefore, the $100(1-\alpha)\%$ two-sided confidence interval for the difference between SNRs based on the parametric bootstrap approach is given by

$$\mathrm{CI}_{\delta.\mathrm{PB}} = [\mathrm{L}_{\delta.\mathrm{PB}}, \mathrm{U}_{\delta.\mathrm{PB}}] = \left[\widehat{\delta} - z_{1-\alpha/2}S^*, \widehat{\delta} + z_{1-\alpha/2}S^*\right], \tag{20}$$

where $z_{1-\alpha/2}$ is the $(1-\alpha/2)$-th quantile of the standard normal distribution and S^* is the standard deviation of $\widehat{\delta}^*$.

The following algorithm is used to construct the parametric bootstrap confidence interval for the difference between SNRs.

Algorithm 5. For a given $\bar{x}^*$, $\bar{y}^*$, s_X^*, and s_Y^*
For g = 1 to m
Compute $\widehat{\theta}_X^*$
Compute $\widehat{\theta}_Y^*$
Compute $\widehat{\delta}^*$
End g loop
Compute S^*
Compute $L_{\delta.PB}$ and $U_{\delta.PB}$

3.2.5 Bayesian approach for the difference between SNRs. For x, let $\sigma_X|x$ be the posterior distribution of σ_X. And let $\mu_X|\sigma_X, x$ be the posterior distribution of μ_X given σ_X. Let $\theta_{X.BS}$ be the posterior distribution using $\sigma_X|x$ and $\mu_X|\sigma_X, x$. Similarly, for y, let $\sigma_Y|y$ be the posterior distribution of σ_Y. And let $\mu_Y|\sigma_Y, y$ be the posterior distribution of μ_Y given σ_Y. Let $\theta_{Y.BS}$ be the posterior distribution using $\sigma_Y|y$ and $\mu_Y|\sigma_Y, y$. Therefore, the posterior distribution of the difference between SNRs is defined by

$$\delta_{BS} = \theta_{X.BS} - \theta_{Y.BS}. \tag{21}$$

The $100(1-\alpha)\%$ two-sided confidence interval for the difference between SNRs based on the Bayesian approach is given by

$$CI_{\delta.BS} = [L_{\delta.BS}, U_{\delta.BS}], \tag{22}$$

where $L_{\delta.BS}$ and $U_{\delta.BS}$ are the lower and upper limits of the shortest $100(1-\alpha)\%$ highest posterior density interval of δ_{BS}, respectively.

The following algorithm is used to construct the Bayesian credible interval for the difference between SNRs.

Algorithm 6. For a given $\bar{x}$, $\bar{y}$, s_X and s_Y
For $g = 1$ to m
Compute $\sigma_X|x$, $\mu_X|\sigma_X, x$, and $\theta_{X.BS}$
Compute $\sigma_Y|y$, $\mu_Y|\sigma_Y, y$, and $\theta_{Y.BS}$
Compute δ_{BS}
End g loop
Compute the shortest $100(1-\alpha)\%$ highest posterior density interval of δ_{BS}

The following algorithm is used to evaluate the coverage probabilities and average lengths of the confidence intervals for the difference between SNRs.

Algorithm 7. For a given μ_X, μ_Y, σ_X, σ_Y and δ
For $h = 1$ to M
Generate x and y
Calculate $\bar{x}$, $\bar{y}$, s_X, and s_Y
Construct the confidence interval $[L_{\delta.GCI}, U_{\delta.GCI}]$
Construct the confidence interval $[L_{\delta.LS}, U_{\delta.LS}]$
Construct the confidence interval $[L_{\delta.MOVER}, U_{\delta.MOVER}]$
Construct the confidence interval $[L_{\delta.PB}, U_{\delta.PB}]$
Construct the confidence interval $[L_{\delta.BS}, U_{\delta.BS}]$
If $L \leq \theta \leq U$, set $p = 1$; else set $p = 0$
Compute $U - L$
End h loop
Compute mean of p defined by the coverage probability
Compute mean of $U - L$ defined by the average length

3.3 Confidence intervals for the common SNR

Consider k independent any distributions with a common SNR γ, let $X_i = (X_{i1}, X_{i2}, \ldots, X_{in_i})$ be a random sample of size n from i-th the distribution with mean μ_i and standard deviation σ_i, where $i = 1, 2, \ldots, k$. Let θ_i be the SNR based on the i-th sample. Moreover, let $\widehat{\theta}_i$ be the maximum likelihood estimator of θ_i. Suppose that $Var(\widehat{\theta}_i)$ is the variance of $\widehat{\theta}_i$.

According to Graybill and Deal (1959), the estimator of the common SNR γ is the weighted average of SNR $\widehat{\theta}_i$ based on k individual samples. The common SNR is defined by

$$\hat{\gamma} = \sum_{i=1}^{k} \frac{\hat{\theta}_i}{\widehat{\mathrm{Var}}\left(\hat{\theta}_i\right)} \Bigg/ \sum_{i=1}^{k} \frac{1}{\widehat{\mathrm{Var}}\left(\hat{\theta}_i\right)}. \tag{23}$$

3.3.1 GCI approach for the common SNR. Following Thangjai and Niwitpong (2020a). Let R_{μ_i} be the GPQ for μ_i and let R_{σ_i} be the GPQ for σ_i. Therefore, the GPQ for θ_i is defined as

$$R_{\theta_i} = \frac{R_{\mu_i}}{R_{\sigma_i}}. \tag{24}$$

The GPQ for the common SNR γ is a weighted average of the GPQ R_{θ_i} based on k individual sample as

$$R_{\gamma} = \sum_{i=1}^{k} \frac{R_{\theta_i}}{R_{\mathrm{Var}\left(\hat{\theta}_i\right)}} \Bigg/ \sum_{i=1}^{k} \frac{1}{R_{\mathrm{Var}\left(\hat{\theta}_i\right)}}, \tag{25}$$

where $R_{\mathrm{Var}(\hat{\theta}_i)}$ is $\mathrm{Var}(\hat{\theta}_i)$ with μ_i replaced by R_{μ_i} and σ_i replaced by R_{σ_i}.

Therefore, the $100(1-\alpha)\%$ two-sided confidence interval for the common SNR based on the GCI approach is given by

$$CI_{\gamma.GCI} = \left[L_{\gamma.GCI}, U_{\gamma.GCI}\right] = \left[R_{\gamma}(\alpha/2), R_{\gamma}(1-\alpha/2)\right], \tag{26}$$

where $R_{\gamma}(\alpha/2)$ and $R_{\gamma}(1-\alpha/2)$ denote the $100(\alpha/2)$-th and $100(1-\alpha/2)$-th percentiles of R_{γ}, respectively.

The following algorithm is used to construct the GCI for the common SNR.

Algorithm 8. For a given $\bar{x}_i$ and s_i
For $g = 1$ to m
Compute R_{μ_i}, R_{σ_i}, and R_{θ_i}
Compute $R_{\mathrm{Var}(\hat{\theta}_i)}$
Compute R_{γ}
End g loop
Compute the $(\alpha/2)$-th quantiles of R_{γ} defined by $R_{\gamma}(\alpha/2)$
Compute the $(1-\alpha/2)$-th quantiles of R_{γ} defined by $R_{\gamma}(1-\alpha/2)$

3.3.2 Adjusted MOVER approach for the common SNR. According to Thangjai and Niwitpong (2019) for $i = 1, 2, \ldots, k$, the variance estimates for $\hat{\theta}_i$ at $\theta_i = l_i$ and $\theta_i = u_i$ are the average variance between these two variances given by

$$\widehat{\mathrm{Var}}\left(\hat{\theta}_i\right) = \frac{\widehat{\mathrm{Var}}\left(\hat{\theta}_{l_i}\right) + \widehat{\mathrm{Var}}\left(\hat{\theta}_{u_i}\right)}{2} = \frac{1}{2}\left(\frac{\left(\hat{\theta}_i - l_i\right)^2}{z_{\alpha/2}^2} + \frac{\left(u_i - \hat{\theta}_i\right)^2}{z_{\alpha/2}^2}\right). \tag{27}$$

As documented by Graybill and Deal (1959), the common SNR is weighted average of the SNR $\hat{\theta}_i$ based on k individual samples. The common SNR is obtained by

$$\hat{\gamma} = \sum_{i=1}^{k} \frac{\hat{\theta}_i}{\widehat{\mathrm{Var}}\left(\hat{\theta}_i\right)} \Bigg/ \sum_{i=1}^{k} \frac{1}{\widehat{\mathrm{Var}}\left(\hat{\theta}_i\right)}, \tag{28}$$

where $\hat{\theta}_i$ is computed by $\hat{\mu}_i/\hat{\sigma}_i$ and $\widehat{\mathrm{Var}}(\hat{\theta}_i)$ is defined as in Equation (27).

According to Krishnamoorthy and Oral (2017), the lower and upper limits of the confidence interval for the common SNR are given by

$$L_{\gamma.AM} = \hat{\gamma} - \sqrt{\sum_{i=1}^{k} \frac{(\hat{\theta}_i - l_i)^2}{(\hat{Var}(\hat{\theta}_{l_i}))^2} \Big/ \sum_{i=1}^{k} \frac{1}{(\hat{Var}(\hat{\theta}_{l_i}))^2}} \tag{29}$$

and

$$U_{\gamma.AM} = \hat{\gamma} + \sqrt{\sum_{i=1}^{k} \frac{(u_i - \hat{\theta}_i)^2}{(\hat{Var}(\hat{\theta}_{u_i}))^2} \Big/ \sum_{i=1}^{k} \frac{1}{(\hat{Var}(\hat{\theta}_{u_i}))^2}}, \tag{30}$$

where

$$\hat{Var}\left(\hat{\theta}_{l_i}\right) = \frac{\left(\hat{\theta}_i - l_i\right)^2}{z_{\alpha/2}^2}$$

and

$$\hat{Var}\left(\hat{\theta}_{u_i}\right) = \frac{\left(u_i - \hat{\theta}_i\right)^2}{z_{\alpha/2}^2}.$$

Suppose that l_i and u_i are the lower and upper limits of the confidence interval for the SNR. Therefore, the $100(1-\alpha)\%$ two-sided confidence interval for the common SNR based on the adjusted MOVER approach is given by

$$CI_{\gamma.AM} = \left[L_{\gamma.AM}, U_{\gamma.AM}\right], \tag{31}$$

where $L_{\gamma.AM}$ is defined as in Equation (29) and $U_{\gamma.AM}$ is defined as in Equation (30).

3.3.3 Computational approach for the common SNR. Following Thangjai and Niwitpong (2020a) the computational approach, introduced by Pal *et al.* (2007), utilizes maximum likelihood estimates. The common SNR, based on the maximum likelihood estimator, is obtained by

$$\hat{\gamma}_{ML} = \sum_{i=1}^{k} \frac{\hat{\theta}_i}{\hat{Var}\left(\hat{\theta}_i\right)} \Big/ \sum_{i=1}^{k} \frac{1}{\hat{Var}\left(\hat{\theta}_i\right)}, \tag{32}$$

where $\hat{\theta}_i = \hat{\mu}_i/\hat{\sigma}_i$ and $\hat{Var}(\hat{\theta}_i)$ is $Var(\hat{\theta}_i)$ with μ_i replaced by $\bar{x}_i$ and σ_i replaced by s_i.

The restricted maximum likelihood estimates (RMLEs) of parameters are used to obtain the computational approach. The maximum likelihood estimates of μ_i, σ_i and γ under $\theta_1 = \theta_2 = \ldots = \theta_k = \gamma$ provide the RMLEs of these parameters. The RMLE of μ_i is defined as $\hat{\mu}_{i(RML)} = \bar{X}_i$. The RMLE of σ_i is defined as $\hat{\sigma}_{i(RML)} = S_i$. And the RMLE of γ is defined as $\hat{\theta}_{i(RML)} = \hat{\mu}_{i(RML)}/\hat{\sigma}_{i(RML)}$.

Let $X_{i(RML)} = (X_{i1(RML)}, X_{i2(RML)}, \ldots, X_{in_i(RML)})$ be artificial sample of size n_i from any distributions with mean $\hat{\mu}_{i(RML)}$ and standard deviation $\hat{\sigma}_{i(RML)}$. For i-th artificial sample, let $\bar{X}_{i(RML)}$ and $S_{i(RML)}$ be the mean and standard deviation of the sample from any distribution.

Let $\overline{x}_{i(RML)}$ and $s_{i(RML)}$ be the observed values of $\overline{X}_{i(RML)}$ and $S_{i(RML)}$, respectively. The common SNR based on k individual samples is obtained by

$$\widehat{\gamma}_{RML} = \sum_{i=1}^{k} \frac{\widehat{\theta}_{i(RML)}}{\widehat{Var}\left(\widehat{\theta}_{i(RML)}\right)} \Bigg/ \sum_{i=1}^{k} \frac{1}{\widehat{Var}\left(\widehat{\theta}_{i(RML)}\right)}, \tag{33}$$

where $\widehat{\theta}_{i(RML)} = \widehat{\mu}_{i(RML)}/\widehat{\sigma}_{i(RML)}$ and $\widehat{Var}(\widehat{\theta}_{i(RML)})$ is $Var(\widehat{\theta}_i)$ with μ_i replaced by $\overline{x}_{i(RML)}$ and σ_i replaced by $s_{i(RML)}$.

Therefore, the $100(1-\alpha)\%$ two-sided confidence interval for the common SNR based on the computational approach is given by

$$CI_{\gamma.CA} = \left[L_{\gamma.CA}, U_{\gamma.CA}\right] = [\widehat{\gamma}_{RML}(\alpha/2), \widehat{\gamma}_{RML}(1-\alpha/2)], \tag{34}$$

where $\widehat{\gamma}_{RML}(\alpha/2)$ and $\widehat{\gamma}_{RML}(1-\alpha/2)$ denote the $100(\alpha/2)$-th and $100(1-\alpha/2)$-th percentiles of $\widehat{\gamma}_{RML}$, respectively.

The following algorithm is used to construct the computational approach for the common SNR.

Algorithm 9. For a given $\overline{x}_i$ and s_i, where $i = 1, 2, \ldots, k$, and common SNR γ
Compute $\widehat{\mu}_{i(RML)} = \overline{x}_i$ and $\widehat{\sigma}_{i(RML)} = s_i$
For $g = 1$ to m
Generate $x_{ij(RML)}$
Compute $\overline{x}_{i(RML)}$ and $s_{i(RML)}$
Compute $\widehat{\gamma}_{RML}$
End g loop
Compute the $(\alpha/2)$-th quantiles of $\widehat{\gamma}_{RML}$ defined by $\widehat{\gamma}_{RML}(\alpha/2)$
Compute the $(1-\alpha/2)$-th quantiles of $\widehat{\gamma}_{RML}$ defined by $\widehat{\gamma}_{RML}(1-\alpha/2)$

3.3.4 Bayesian approach for the common SNR. For $i = 1, 2, \ldots, k$, let $\sigma_i|x_i$ be the posterior distribution of σ_i. And let $\mu_i|\sigma_i, x_i$ be the posterior distribution of μ_i given σ_i. Let θ_i be the posterior distribution using $\sigma_i|x_i$ and $\mu_i|\sigma_i, x_i$. Let $Var(\theta_i)$ be the variance of θ_i using $\sigma_i|x_i$ and $\mu_i|\sigma_i, x_i$.

The common SNR is defined by

$$\gamma_{BS} = \sum_{i=1}^{k} \frac{\theta_i}{Var(\theta_i)} \Bigg/ \sum_{i=1}^{k} \frac{1}{Var(\theta_i)}. \tag{35}$$

The $100(1-\alpha)\%$ two-sided confidence interval for the common SNR based on the Bayesian approach is given by

$$CI_{\gamma.BS} = \left[L_{\gamma.BS}, U_{\gamma.BS}\right], \tag{36}$$

where $L_{\gamma.BS}$ and $U_{\gamma.BS}$ are the lower and upper limits of the shortest $100(1-\alpha)\%$ highest posterior density interval of γ_{BS}, respectively.

The following algorithm is used to construct the Bayesian credible interval for the common SNR.

Algorithm 10. For a given $\overline{x}_i$ and s_i
For $g = 1$ to m
Compute $\sigma_i|x_i$, $\mu_i|\sigma_i, x_i$, and θ_i
Compute $Var(\theta_i)$
Compute γ_{BS}

End g loop
Compute the shortest $100(1-\alpha)\%$ highest posterior density interval of γ_{BS}

The following algorithm is used to evaluate the coverage probabilities and average lengths of the confidence intervals for the common SNR.

Algorithm 11. For a given μ_i, σ_i, and γ, where $i = 1, 2, \ldots, k$
For $h = 1$ to M
Generate x_{ij}, where $i = 1, 2, \ldots, k$ and $j = 1, 2, \ldots, n_i$
Calculate $\bar{x}_i$ and s_i
Construct the confidence interval $[L_{\gamma.GCI}, U_{\gamma.GCI}]$
Construct the confidence interval $[L_{\gamma.AM}, U_{\gamma.AM}]$
Construct the confidence interval $[L_{\gamma.CA}, U_{\gamma.CA}]$
Construct the confidence interval $[L_{\gamma.BS}, U_{\gamma.BS}]$
If $L \leq \theta \leq U$, set $p = 1$; else set $p = 0$
Compute $U - L$
End h loop
Compute mean of p defined by the coverage probability
Compute mean of $U - L$ defined by the average length

4. Simulation studies

A simulation study was conducted to assess the coverage probabilities and average lengths of the confidence intervals using the R statistical program. The criteria for selecting the best-performing confidence interval were a coverage probability greater than or equal to $1-\alpha$ and the shortest average length for each tested scenario.

For the SNR simulation study, the sample size, population mean, population standard deviation, and population SNR were set. For each set of parameters, M random samples were generated. For the GCI and Bayesian approach, m times of R_θ and θ_{BS} were obtained for each of the random samples. In this simulation study, the sample sizes were $n = 30, 50, 100$; the population mean was $\mu = 1$; the SNR $\theta = 1, 2, 5, 10$; and the population standard deviation was computed as $\sigma = \sqrt{\log((1/\theta^2) + 1)}$.5,000 random samples were generated for each set of parameters. For the GCI and Bayesian approaches, 2,500 R_θ's and 2,500 θ_{BS}'s were obtained for each of the random samples. Table 1 presents the coverage probabilities and average lengths of the 95% two-sided confidence intervals for the SNR of the log-normal distribution, utilizing the GCI, large sample, and Bayesian approaches. The findings indicate that the coverage probabilities of all approaches closely align with the nominal confidence level of 0.95. Overall, the Bayesian approach emerged as the best approach, excelling in both coverage probability and average length. Notably, the GCI approach outperforms the large sample and Bayesian approaches in terms of both coverage probability and average length.

For the difference between SNRs simulation study, the sample sizes, population means, population standard deviations, and population SNRs were set. For each set of parameters, M random samples were generated. For the GCI, parametric bootstrap, and Bayesian approaches, m times of R_δ, $\widehat{\delta}^*$, δ_{BS} were obtained for each of the random samples. In this simulation study, the sample sizes were $(n, m) = (30, 30), (30, 50), (50, 50), (50, 100), (100, 100)$; the population means were $(\mu_X, \mu_Y) = (1,1)$; the population SNRs were $(\theta_X, \theta_Y) = (10, 1), (10, 2), (10, 5), (10, 10)$; and the population standard deviations were computed as $\sigma_X = \sqrt{\log((1/\theta_X^2) + 1)}$ and $\sigma_Y = \sqrt{\log((1/\theta_Y^2) + 1)}$. The coverage probabilities and average lengths of the 95% two-sided confidence intervals for the difference between the SNRs of the log-normal distributions were evaluated based on 5,000 replications, and 2,500 R_δ's, 2,500 $\widehat{\delta}^*$, and 2,500 δ_{BS} were obtained for the GCI, parametric bootstrap, and Bayesian approaches. The results are displayed in Table 2, showing that all approaches were satisfactory for all cases, except for the parametric bootstrap approach, which demonstrates a coverage probability lower than the nominal confidence level of 0.95. The GCI approach surpasses the others in terms of both coverage probability and average length.

For the common SNR simulation study, the sample cases, sample sizes, population means, population standard deviations, and population common SNR were set. For each set of parameters, M random samples were generated. For the GCI, computational, and Bayesian approaches, m times of R_γ, $\widehat{\gamma}_{RML}$, and γ_{BS} were obtained for each of the random samples. In this simulation study, the sample cases were $k = 3$; the sample sizes were $(n_1, n_2, n_3) = (30,30,30)$, (50,50,50), (30,50,100), (100,100,100); the population means were $(\mu_1, \mu_2, \mu_3) = (1,1,1)$; the population standard deviations were $(\sigma_1, \sigma_2, \sigma_3) = (0.10,0.29,0.47)$, (0.29,0.47,0.83). For each set of parameters, 5,000 random samples were generated. For the GCI, computational, and Bayesian approaches, 1,000 R_γ's and 1,000 $\widehat{\gamma}_{RML}$'s, and 1,000 γ_{BS} were obtained for each of the random samples. The results are showcased in Table 3, revealing that the GCI and Bayesian approaches were superior in terms of coverage probability. Additionally, the computational approach demonstrated coverage probabilities close to the nominal confidence level of 0.95 for large sample sizes. However, the adjusted MOVER approach exhibited coverage probabilities lower than the nominal confidence level of 0.95. Overall, the GCI approach emerged as the best approach, excelling in both coverage probability and average length.

The R code for computing the coverage probabilities and average lengths of 95% two-sided confidence intervals for the common SNR of several log-normal distributions using the GCI, adjusted MOVER, computational, and Bayesian approaches is presented in the Appendix.

5. Empirical results

Changes in stock market indices and their constituents occur over time due to market dynamics and the criteria set by the exchange. In the context of Thailand, SET50, SET100, and sSET are notable indices. The SET50 Index mirrors the price fluctuations of 50 large-capitalization securities with substantial trading liquidity on the Stock Exchange of

Table 1. The coverage probabilities (CP) and average lengths (AL) of 95% two-sided confidence intervals for the SNR of log-normal distribution

n	θ	CP (AL) $CI_{\theta.GCI}$	$CI_{\theta.LS}$	$CI_{\theta.BS}$
30	1	0.9486 (0.7296)	0.9490 (0.7302)	0.9478 (0.7243)
	2	0.9506 (1.1787)	0.9516 (1.1815)	0.9462 (1.1691)
	5	0.9452 (2.6796)	0.9460 (2.6927)	0.9424 (2.6563)
	10	0.9522 (5.2743)	0.9550 (5.3010)	0.9498 (5.2268)
50	1	0.9498 (0.5552)	0.9498 (0.5554)	0.9488 (0.5510)
	2	0.9478 (0.8957)	0.9482 (0.8968)	0.9458 (0.8887)
	5	0.9472 (2.0433)	0.9480 (2.0498)	0.9444 (2.0263)
	10	0.9500 (4.0332)	0.9510 (4.0460)	0.9496 (3.9995)
100	1	0.9520 (0.3888)	0.9528 (0.3888)	0.9490 (0.3856)
	2	0.9536 (0.6249)	0.9540 (0.6252)	0.9532 (0.6202)
	5	0.9522 (1.4279)	0.9516 (1.4295)	0.9478 (1.4166)
	10	0.9488 (2.8132)	0.9508 (2.8180)	0.9490 (2.7917)

Source(s): Authors' calculation

Table 2. The CPs and ALs of 95% two-sided confidence intervals for the difference between SNRs of log-normal distributions

(n, m)	(θ_X, θ_Y)	CP (AL) $CI_{\delta.GCI}$	$CI_{\delta.LS}$	$CI_{\delta.MOVER}$	$CI_{\delta.PB}$	$CI_{\delta.BS}$
(30,30)	(10,1)	0.9506 (5.3335)	0.9518 (5.3605)	0.9602 (5.5937)	0.9392 (5.6483)	0.9492 (5.2891)
	(10,2)	0.9516 (5.4272)	0.9528 (5.4499)	0.9608 (5.6870)	0.9410 (5.7394)	0.9478 (5.3764)
	(10,5)	0.9512 (5.9269)	0.9530 (5.9553)	0.9618 (6.2144)	0.9474 (6.3042)	0.9482 (5.8786)
	(10,10)	0.9548 (7.5176)	0.9578 (7.5454)	0.9660 (7.8737)	0.9470 (8.0065)	0.9562 (7.4591)
(30,50)	(10,1)	0.9484 (5.2930)	0.9504 (5.3196)	0.9588 (5.5500)	0.9400 (5.6141)	0.9452 (5.2461)
	(10,2)	0.9474 (5.3725)	0.9488 (5.3991)	0.9588 (5.6312)	0.9396 (5.6993)	0.9460 (5.3268)
	(10,5)	0.9484 (5.6744)	0.9500 (5.6973)	0.9584 (5.9316)	0.9410 (6.0113)	0.9472 (5.6214)
	(10,10)	0.9568 (6.6862)	0.9572 (6.7086)	0.9642 (6.9558)	0.9500 (7.0183)	0.9554 (6.6342)
(50,50)	(10,1)	0.9448 (4.0614)	0.9456 (4.0721)	0.9518 (4.1751)	0.9340 (4.1708)	0.9410 (4.0268)
	(10,2)	0.9488 (4.1327)	0.9498 (4.1439)	0.9550 (4.2488)	0.9376 (4.2406)	0.9460 (4.0970)
	(10,5)	0.9472 (4.5328)	0.9474 (4.5449)	0.9530 (4.6600)	0.9452 (4.6654)	0.9430 (4.4986)
	(10,10)	0.9498 (5.7174)	0.9508 (5.7315)	0.9560 (5.8766)	0.9454 (5.8767)	0.9486 (5.6727)
(50,100)	(10,1)	0.9488 (4.0481)	0.9506 (4.0596)	0.9576 (4.1619)	0.9392 (4.1476)	0.9466 (4.0145)
	(10,2)	0.9466 (4.0751)	0.9480 (4.0896)	0.9536 (4.1919)	0.9428 (4.1858)	0.9426 (4.0444)
	(10,5)	0.9534 (4.2789)	0.9520 (4.2896)	0.9600 (4.3915)	0.9424 (4.3751)	0.9514 (4.2441)
	(10,10)	0.9488 (4.9270)	0.9504 (4.9364)	0.9542 (5.0405)	0.9426 (5.0405)	0.9482 (4.8904)
(100,100)	(10,1)	0.9504 (2.8447)	0.9508 (2.8476)	0.9542 (2.8828)	0.9418 (2.8633)	0.9460 (2.8196)
	(10,2)	0.9462 (2.8867)	0.9488 (2.8885)	0.9514 (2.9242)	0.9432 (2.9072)	0.9452 (2.8609)
	(10,5)	0.9484 (3.1648)	0.9470 (3.1688)	0.9500 (3.2080)	0.9460 (3.1970)	0.9464 (3.1406)
	(10,10)	0.9516 (3.9844)	0.9532 (3.9880)	0.9558 (4.0373)	0.9494 (4.0215)	0.9494 (3.9529)

Source(s): Authors' calculation

Thailand. Similarly, the SET100 Index encompasses the price movements of 100 large-capitalization securities with notable trading liquidity on the same exchange. On the other hand, the sSET Index captures the price changes of common stocks beyond those included in the SET50 and SET100 indices. These stocks exhibit consistent liquidity and adhere to specified requirements related to share distribution among minor shareholders.

Price-earnings ratios for the SET50, SET100, and sSET indexes are computed from monthly index data provided by the Stock Exchange of Thailand. This study focuses on monthly index data spanning from January to November 2023, as detailed in Table 4. The histograms depicting daily rainfall data can be found in Figure 1, and Table 5 presents sample sizes, means, standard deviations, and SNRs for the three indexes. Before applying our methods to real data, it is crucial to assess the assumption that the logarithms of the data are drawn from a normal distribution. Traditionally, the Shapiro–Wilk normality test was employed, yielding p-values of 0.3922, 0.1467, and 0.08508 for SET50, SET100, and sSET indexes, respectively. Recognizing the limitations of p-values in testing, alternative methods for checking normality include graphical tools such as QQ-plots or Bayesian tests. Analysis of Table 6 reveals the minimum Akaike Information Criterion (AIC) values from Bayesian tests across five regions, indicating that SET50, SET100, and sSET indexes follow log-normal distributions. Furthermore, the normal QQ-plots of log-data in Figure 2 affirm the results of the Bayesian test. For illustrative purposes, we exclusively select data from log-normal distributions to showcase our estimation approaches.

For SET50 index, the 95% confidence intervals for the SNR based on the GCI, large sample, and Bayesian approaches are $CI_{\theta.GCI} = [11.4901, 28.5148]$ with an interval length of 17.0247, $CI_{\theta.LS} = [11.2126, 28.7475]$ with an interval length of 17.5349, and $CI_{\theta.BS} = [10.8550, 28.0229]$ with an interval length of 17.1679, respectively. For SET100 index, the 95% confidence intervals for the SNR based on the GCI, large sample, and Bayesian approaches are $CI_{\theta.GCI} = [8.6529, 21.7304]$ with an interval length of 13.0775, $CI_{\theta.LS} = [8.4493, 21.6850]$ with an interval length of 13.2357, and $CI_{\theta.BS} = [8.0436, 21.1895]$ with an interval length of 13.1459, respectively. For sSET index, the 95% confidence intervals for the SNR based on the GCI, large sample, and Bayesian approaches are $CI_{\theta.GCI} = [7.9635, 20.0325]$ with an interval length of 12.0690, $CI_{\theta.LS} = [7.8989, 20.2794]$ with an interval length of 12.3805, and $CI_{\theta.BS} = [7.9815, 20.1948]$ with an interval length of 12.2133, respectively. Notably, the confidence intervals for the SNR based on the GCI, large sample, and Bayesian approaches encompass the true value of the SNR. However, the GCI approach has a shorter length than the large sample and Bayesian approaches.

For difference between SET50 index and SET100 index, the true difference between the SNRs is 4.9129. The 95% confidence intervals for the difference between SNRs based on the GCI, large sample, MOVER, parametric bootstrap, Bayesian approaches are $CI_{\delta.GCI} =$

Table 3.
The CPs and ALs of 95% two-sided confidence intervals for the common SNR of several log-normal distributions

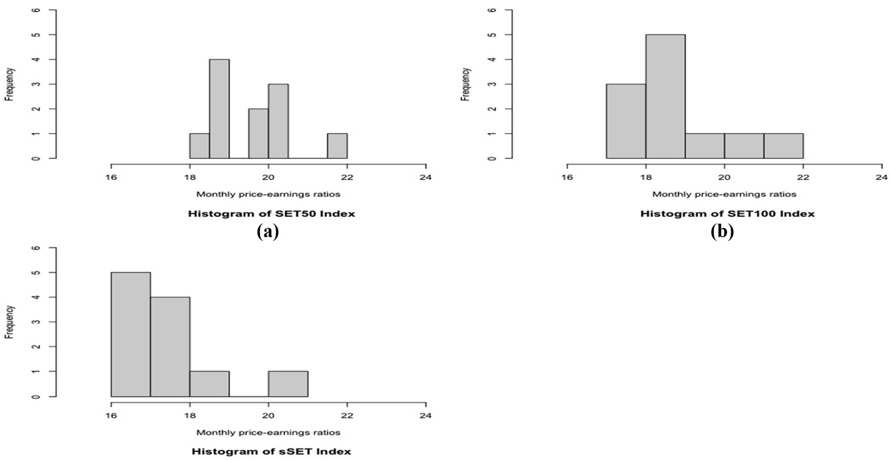
		CP (AL)			
(n_1, n_2, n_3)	$(\sigma_1, \sigma_2, \sigma_3)$	$CI_{\gamma.GCI}$	$CI_{\gamma.AM}$	$CI_{\gamma.CA}$	$CI_{\gamma.BS}$
(30,30,30)	(0.10,0.29,0.47)	0.9530 (1.2606)	0.8846 (1.0129)	0.9394 (1.2970)	0.9478 (1.2424)
	(0.29,0.47,0.83)	0.9484 (0.7425)	0.8908 (0.6071)	0.9386 (0.7596)	0.9444 (0.7317)
(50,50,50)	(0.10,0.29,0.47)	0.9512 (0.9656)	0.8814 (0.7612)	0.9456 (0.9823)	0.9504 (0.9523)
	(0.29,0.47,0.83)	0.9550 (0.5699)	0.8952 (0.4578)	0.9498 (0.5776)	0.9528 (0.5614)
(30,50,100)	(0.10,0.29,0.47)	0.9482 (0.6554)	0.9100 (0.5783)	0.9426 (0.6650)	0.9426 (0.6465)
	(0.29,0.47,0.83)	0.9484 (0.4010)	0.9170 (0.3554)	0.9508 (0.4065)	0.9444 (0.3954)
(100,100,100)	(0.10,0.29,0.47)	0.9544 (0.6780)	0.8736 (0.5262)	0.9458 (0.6842)	0.9494 (0.6689)
	(0.29,0.47,0.83)	0.9448 (0.4000)	0.8792 (0.3169)	0.9454 (0.4032)	0.9414 (0.3944)

Source(s): Authors' calculation

Index	Price-earnings ratios				
SET50	18.90	20.09	20.07	18.71	18.90
	18.79	19.69	21.77	20.45	19.58
	18.33				
SET100	17.54	18.79	18.80	17.52	18.24
	18.07	18.66	21.73	20.44	19.43
	17.87				
sSET	16.00	17.52	17.49	16.45	17.15
	16.07	16.15	20.05	18.84	17.20
	16.44				

Source(s): Stock Exchange of Thailand (https://www.set.or.th/th/market/statistics/market-statistics/main) Authors' calculation

Table 4. Price-earnings ratios of three indexes



Source(s): Author's calculation

Figure 1. Histogram plots of monthly price-earnings ratios of three indexes

Sample statistics	SET50	Index SET100	sSET
n_i	11	11	11
$\overline{y}_i$	19.57	18.83	17.21
s_{Y_i}	1.00	1.29	1.26
$\overline{x}_i$	2.97	2.93	2.84
s_{X_i}	0.05	0.07	0.07
$\widehat{\theta}_i$	19.98	15.07	14.09

Source(s): Authors' calculation

Table 5. Sample statistics of price-earnings ratios of three indexes

$[-5.9406, 15.6047]$ with a length of interval of 21.5453, $CI_{\delta.LS} = [-6.0718, 15.8977]$ with a length of interval of 21.9695, $CI_{\delta.MOVER} = [-7.5748, 17.4007]$ with a length of interval of 24.9755, $CI_{\delta.PB} = [-11.6552, 21.9753]$ with a length of interval of 33.6305, and $CI_{\delta.BS} = [-6.2144, 15.6602]$ with a length of interval of 21.8746, respectively. For difference between SET50 index and sSET index, the true difference between the SNRs is 5.8909. The 95% confidence intervals for the difference between SNRs based on the GCI, large sample, MOVER, parametric bootstrap, Bayesian approaches are $CI_{\delta.GCI} = [-4.1340, 16.4281]$ with a length of interval of 20.5621, $CI_{\delta.LS} = [-4.8416, 16.6235]$ with a length of interval of 21.4651, $CI_{\delta.MOVER} = [-6.3101, 18.0920]$ with a length of interval of 24.4021, $CI_{\delta.PB} = [-9.7428, 21.7973]$ with a length of interval of 31.5401, and $CI_{\delta.BS} = [-5.2779, 15.4926]$ with a length of interval of 20.7705. For difference between SET100 index and sSET index, the true difference between the SNRs is 0.9780. The 95% confidence intervals for the difference between SNRs based on the GCI, large sample, MOVER, parametric bootstrap, Bayesian approaches are $CI_{\delta.GCI} = [-7.7565, 9.7044]$ with a length of interval of 17.4609, $CI_{\delta.LS} = [-8.0837, 10.0398]$ with a length of interval of 18.1235, $CI_{\delta.MOVER} = [-9.3236, 11.2796]$ with a length of interval of 20.6032, $CI_{\delta.PB} = [-14.3997, 16.7647]$ with a length of interval of 31.1644, and $CI_{\delta.BS} = [-8.0435, 9.8604]$ with a length of interval of 17.9039. The results indicate that all confidence intervals contain the true difference between the SNRs. However, the GCI approach stands out by providing the shortest length, making it the most preferable among the alternatives.

The true common SNRs is 15.6870. The 95% confidence intervals for the common SNR based on GCI, adjusted MOVER, computational, and Bayesian approaches are $CI_{\gamma.GCI} =$

Table 6.
The AIC values of price-earnings ratios of three indexes

Distribution	AIC SET50 index	SET100 index	sSET index
Normal	34.09	39.76	39.31
Log-normal	33.67	39.00	38.50
Gamma	33.88	39.34	38.86
Exponential	88.43	87.58	85.61

Source(s): Authors' calculation

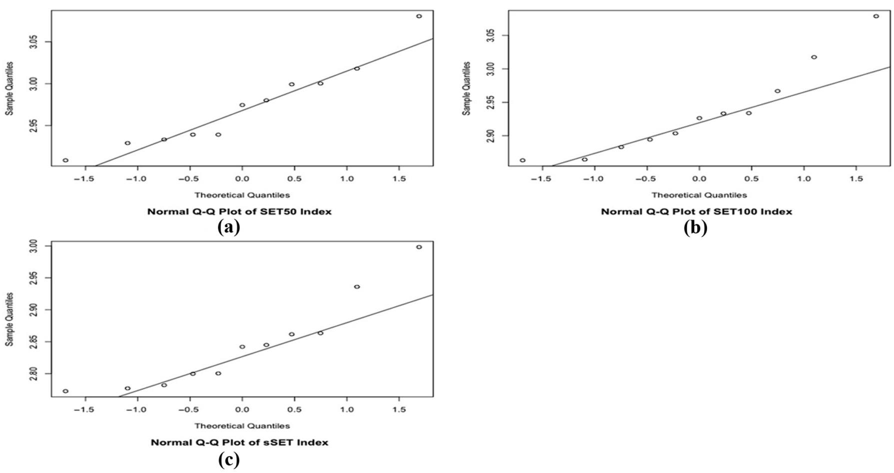


Figure 2.
The normal QQ-plots of log-monthly price-earnings ratios of three indexes

Source(s): Author's calculation

[9.7447,18.5943] with a length of interval of 8.8496, $CI_{\gamma.AM} = [11.1191,20.2548]$ with a length of interval of 9.1357, $CI_{\gamma.CA} = [12.0582,21.0089]$ with a length of interval of 8.9507, and $CI_{\gamma.BS} =$ [9.7514,18.7842] with a length of interval of 9.0328. The findings suggest that all confidence intervals include the true common SNR, with the GCI approach having a shorter length compared to the others.

6. Conclusion

The GCI approach showed better results than other techniques in terms of coverage probability and average length for both the SNR and the difference between SNRs, with the Bayesian approach performing similarly to the GCI approach. Therefore, it is recommended to use the GCI approach for constructing confidence intervals for these parameters. Regarding the common SNR, the Bayesian approach had the shortest average length. Hence, it is recommended to use the Bayesian approach for constructing confidence intervals for the common SNR.

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Appendix

The supplementary material for this article can be found online.

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